The PQ-duality between the trigonometric Calogero model and the rational Ruijsenaars model as a spectral duality

One of the most interesting directions in the study of integrable systems is the search for possible dualities between integrable models. Two well-known examples of such dualities are the spectral duality and the Ruijsenaars (or PQ-) duality. Spectral duality describes the duality between two spin systems that have a Lax representation that depends on the spectral parameter z. The spectral curves for such dual systems coincide; in particular, the trigonometric Gaudin model and the Heisenberg chain turn out to be spectral dual [1]. The Ruijsenaars duality for N-particle systems builds a correspondence between the coordinate variables of one system and the action variables of another. Such a duality is known, for example, between the trigonometric Calogero model and the rational Ruijsenaars-Schneider model [2]. We have shown that PQ-duality for these models can be described in terms of spectral duality. In particular, one can make a gauge transformation that adds dependence on the spectral parameter to the Lax matrix of the trigonometric Calogero system without changing the Lax equation. Having done such a transformation, we have shown that the resulting Lax matrix can be described as a degenerate Gaudin type Lax matrix. The spectrally dual system after the inverse gauge transformation of the Lax matrix turns into the Lax matrix of the PQ-dual rational Ruijsenaars-Schneider model. Hense we can consider PQ-duality   as   a   special   case   of   spectral   duality.

[1] Mironov A., Morozov A., Runov B., Zenkevich Y., Zotov A., Spectral Duality Between Heisenberg Chain and Gaudin Model // Lett Math Phys 2013 V. 103 P. 299–329.
[2] Ruijsenaars S., Action-angle maps and scattering theory for some finite-dimensional integrable systems III. Sutherland type systems and their duals // Publ. RIMS 1995, v. 31 pp. 247–353.

Elizaveta Trunina

MIPT and MI RAS

yelizaveta.kupcheva@phystech.edu