# Nonlinear dynamics of slipping flows

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## Abstract

The process of breaking of inviscid incompressible flows along a rigid body with slipping boundary conditions is studied. Such slipping flows may be considered compressible on the rigid surface, where the normal velocity vanishes. It is the main reason for the formation of a singularity for the gradient of the velocity component parallel to rigid border. Slipping flows are studied analytically in the framework of two- and three-dimensional inviscid Prandtl equations. Criteria for a gradient catastrophe are found in both cases. For 2D Prandtl equations breaking takes place both for the parallel velocity along the boundary and for the vorticity gradient. For three-dimensional Prandtl flows, breaking, i.e. the formation of a fold in a finite time, occurs for the symmetric part of the velocity gradient tensor, as well as for the antisymmetric part — vorticity. Simultaneously it leads to the formation of jets in perpendicular direction to the boundary that together with the vorticity blowup can be considered as the tornado type generation.

The problem of the formation of velocity gradients for flows between two parallel plates is studied numerically in the framework of two-dimensional Euler equations. It is shown that the maximum velocity gradient grows exponentially with time on a rigid boundary with a simultaneous increase in the vorticity gradient according to a double exponential law. Careful analysis shows that this process is nothing more than the folding, with a power-law relationship between the maximum velocity gradient and its width: $\max|u_x| \propto \ell^{-2/3}$.

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[1] E.A. Kuznetsov, E.A. Mikhailov, Annals of Physics **447**, 169088 (2022).