Topological quantum computation via edge states in disordered honeycomb Kitaev model

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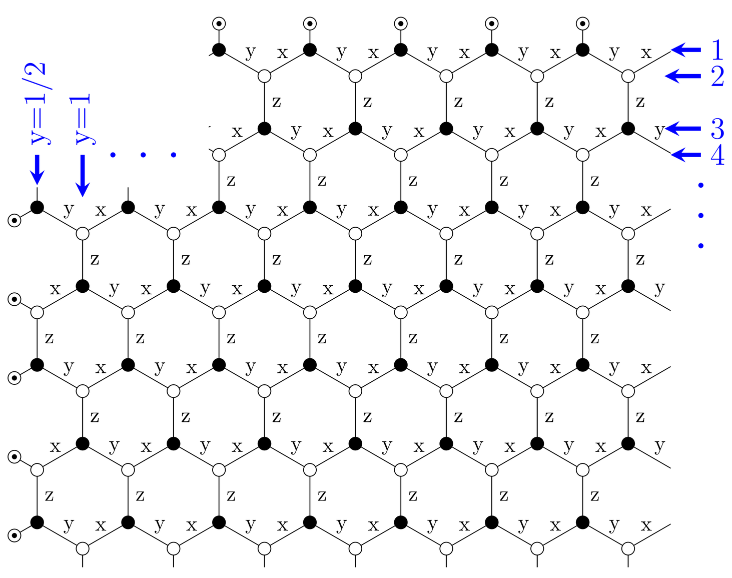
Honeycomb Kitaev model - edge states

The Hamiltonian of the Kitaev honeycomb spin model [A. Kitaev, Ann. Phys. (2006)]

$$H = -J_x \sum_{x-\text{links}} \sigma_x^i \sigma_x^j - J_y \sum_{y-\text{links}} \sigma_y^i \sigma_y^j - J_z \sum_{z-\text{links}} \sigma_z^i \sigma_z^j - \mathbf{h} \sum_j \sigma^j$$



Fermionic representation

$$\sigma_x^j = i b_x^j c^j, \quad \sigma_y^j = i b_y^j c^j, \quad \sigma_z^j = i b_z^j c^j$$

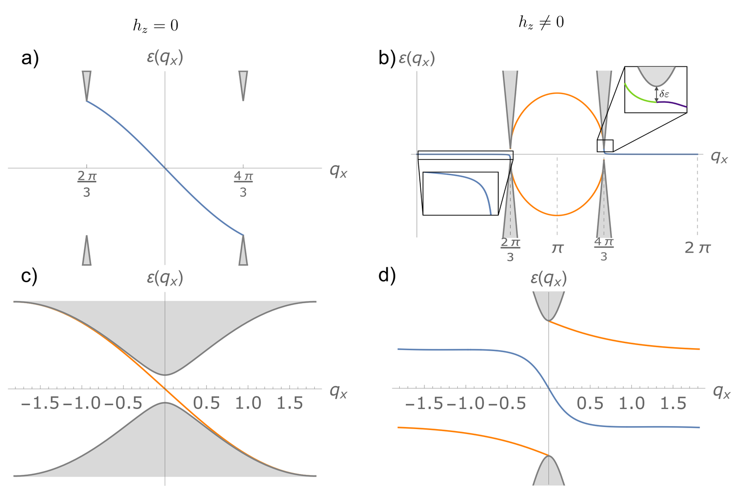
The third-order contribution of the magnetic field $V^{(3)} = -\kappa \sum_{jkl} \sigma_x^j \sigma_y^k \sigma_z^l$, $\kappa \propto h^3/J^2$

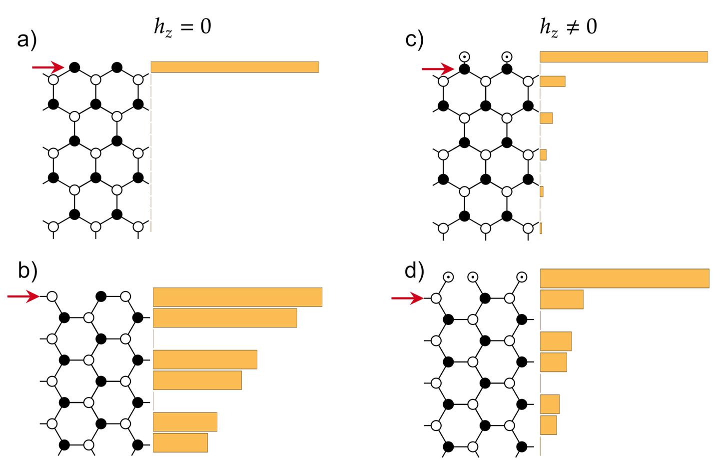
Fermionized Hamiltonian

$$H = \frac{1}{2} \sum_{\mathbf{q}} A(\mathbf{q})_{\lambda\mu} c_{-\mathbf{q}\lambda} c_{\mathbf{q}\mu}, \quad A(\mathbf{q}) = \begin{pmatrix} \Delta(\mathbf{q}) & if(\mathbf{q}) \\ -if(-\mathbf{q}) & -\Delta(\mathbf{q}) \end{pmatrix}, \quad f(\mathbf{q}) = 2J\left(e^{i\mathbf{q}\mathbf{n}_1} + e^{i\mathbf{q}\mathbf{n}_2} + 1\right)$$

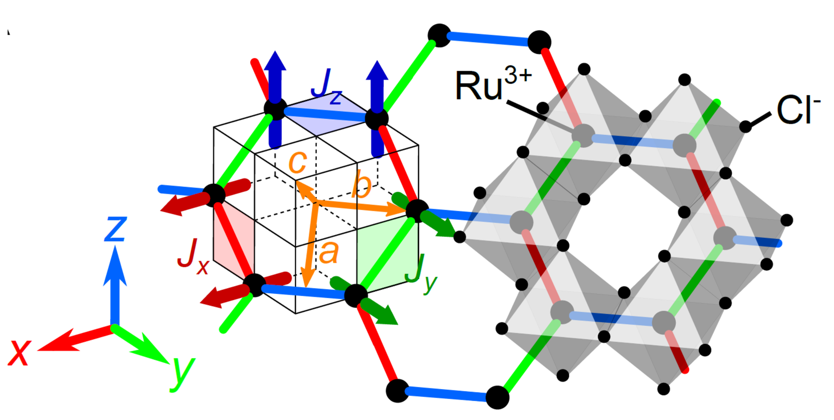
Edge mode spectrum for **zigzag** edge and group velocity and the **armchair** edge

$$\varepsilon^a(q_x) = 12\kappa \sin q_x, \quad \varepsilon^b(q_x) = -\frac{h_z^2 \kappa}{J^2} \frac{\sin q_x + \tan \frac{q_x}{2}}{\cos^2 \frac{q_x}{2} - \frac{1}{4} + \frac{h_z^2}{4J^2}}, \quad v_g^c = -\sqrt{3}J, \quad v_g^d = -\frac{2\sqrt{3}Jh_b^2}{2h_b^2 + \sqrt{3}|\Delta| J}$$

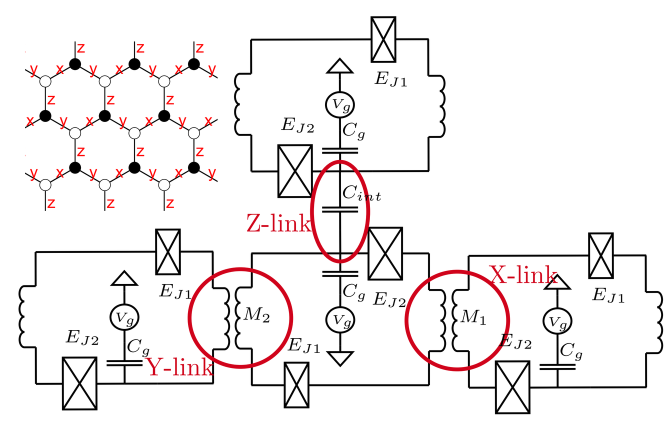




Honeycomb Kitaev model - possible realisations

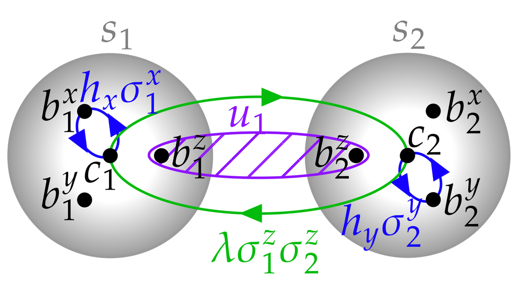


[T. Shibauchi et al, APS Mar. Meet. Abs. (2021)]



[J. Q. You et al, Phys. Rev. B (2010)]

Two-qubit operations

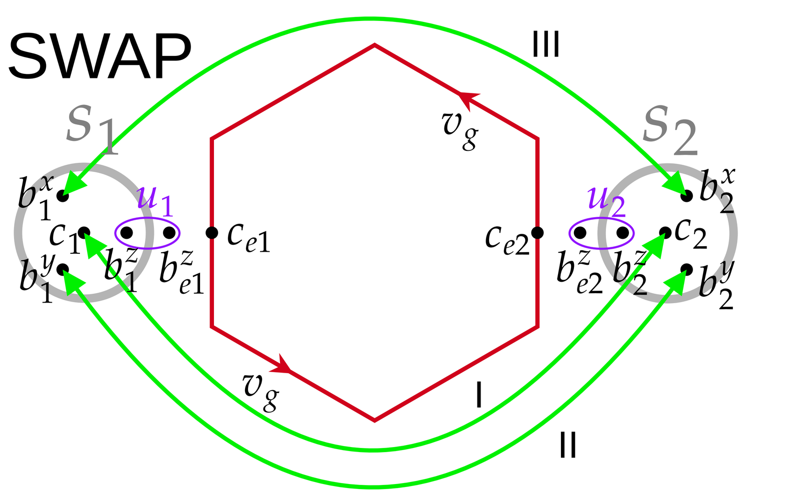


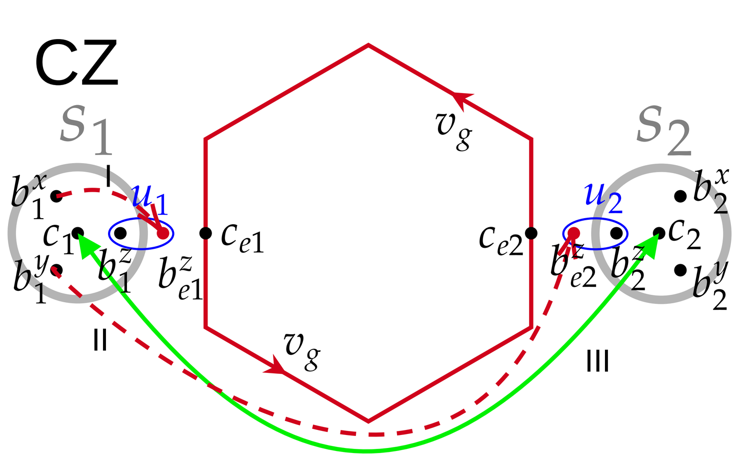
External magnetic field

$$H_{h_\alpha} = h_\alpha \sigma_\alpha = i h_\alpha b_\alpha c \to \begin{matrix} b_\alpha\left(t^* = \frac{\pi}{4h_\alpha}\right) = c(0) \\ c(t^*) = -b_\alpha(0) \end{matrix}$$

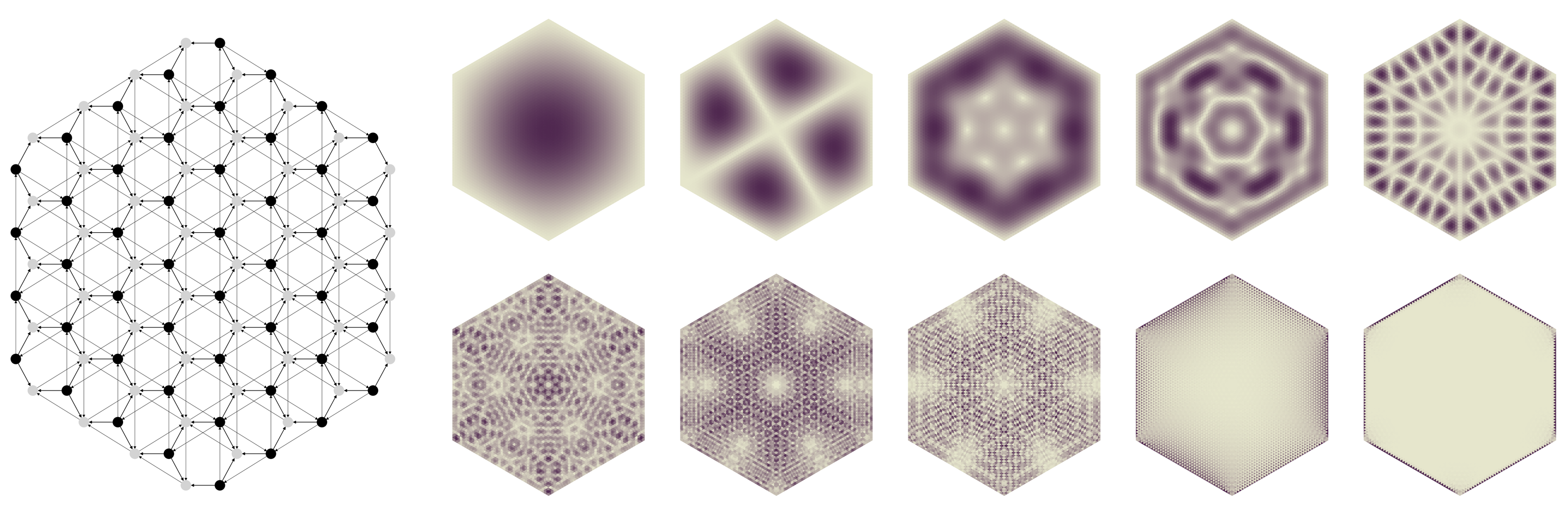
$\sigma_z\sigma_z$-interaction

$$H_\lambda = -\lambda \sigma_z^1 \sigma_z^2 = \lambda \hat{u}^{12} i c^1 c^2 \to \begin{matrix} c^1(t^*) = \hat{u}^{12} c^2(0) \\ c^2(t^*) = -\hat{u}^{12} c^1(0) \end{matrix}$$





Eigenstates in a finite sample



The Hamiltonian in graph representation and several examples of bulk (1-8) states with energies between $6J$ and $3.4J$ and two edge eigenstates (9-10) with energies $-0.25J$ and $-0.005J$. $\kappa = 0.054$

Writing operation - point-like method

State transfer between external qubits

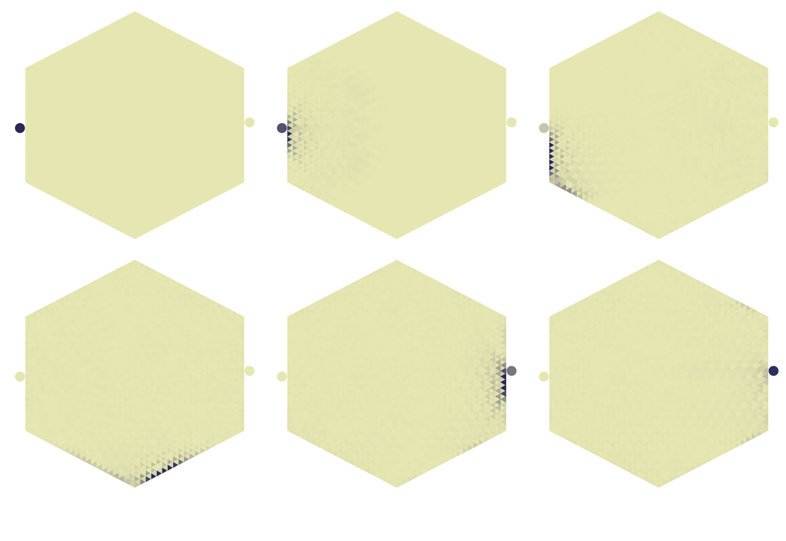
$$c_{ext} \mapsto u_g \sum_i \alpha_i c_{edi} \mapsto P u_g u_g' c_{ext}' + \sqrt{1-P^2} d_{ed}'$$

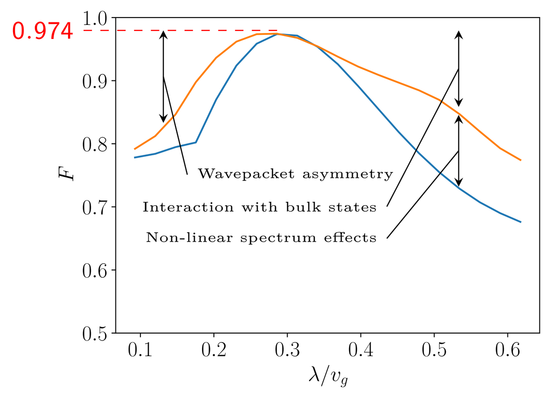
For SWAP operation ($\sigma_{ext}^\alpha = i b_{ext}^\alpha c_{ext}$)

$$\sigma_{ext}^\alpha \mapsto P^2 (\sigma_{ext}')^\alpha + P\sqrt{1-P^2} u_g u_g' (c' d_{b^\alpha} + b'^\alpha d_c) + (1-P^2) d_c d_{b^\alpha}.$$

The fidelity of the operation

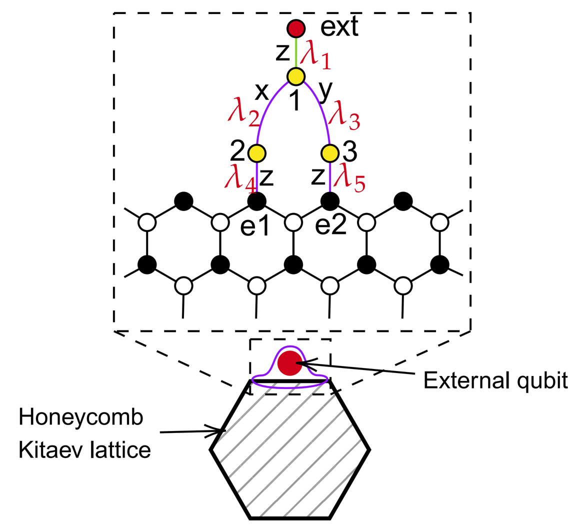
$$F = \frac{1+P^2}{2}.$$





Numerical simulations of the evolution show for point-like weak constant coupling, $\lambda \ll v_{gr}$, we obtained fidelity higher than 97% of the state transfer between external qubits.

Writing operation - tree-like method



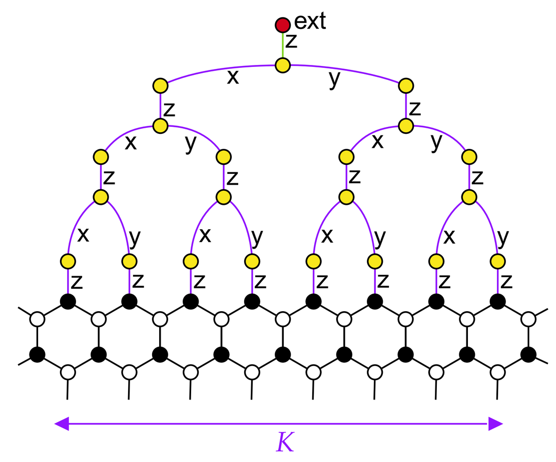
First step - a pulse of the λ_1 coupling for a period $t_1 = \frac{\pi}{4\lambda_1}$: $c_{ext} \mapsto u_g c_1$.

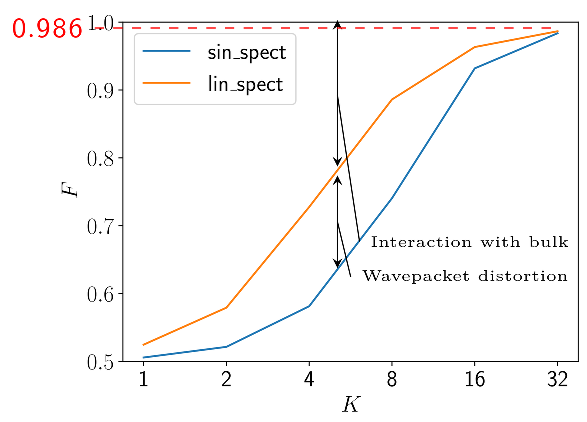
Second step, the couplings λ_2 and λ_3 are switched on:

$$H = i\lambda_2 c_1 c_2 + i\lambda_3 c_1 c_3, \quad c_{ext} \mapsto \frac{u_g}{\sqrt{\lambda_2^2 + \lambda_3^2}} (\lambda_2 c_2 + \lambda_3 c_3).$$

Proceeding similarly in the following row, we map the initial c_{ext} to $\frac{u_g}{\sqrt{\lambda_2^2+\lambda_3^2}}(\lambda_2 c_{ed1} + \lambda_3 c_{ed2})$.

For deeper tree-like structures, wider and hence less dispersing wave packets are created, which improves the fidelity F.





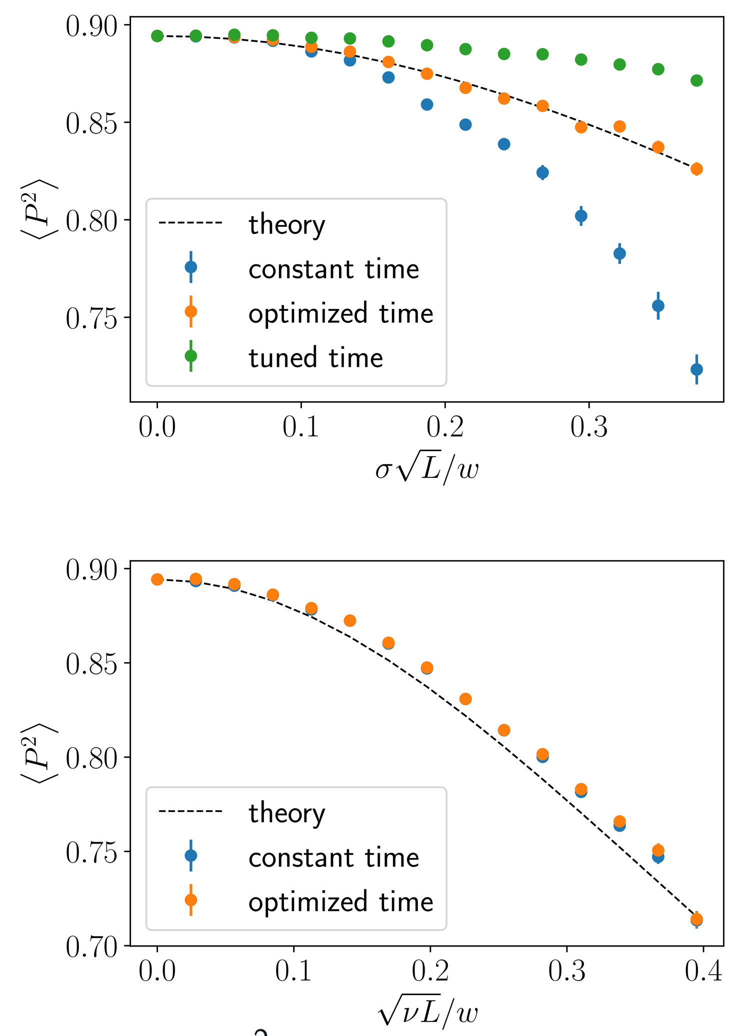
Disorder and fidelity

The Hamiltonian of the disordered system

$$H = H_{\text{Kit}} + V_{\text{dis}}$$

The first order correction to the edge spectrum changes $v_g = \delta\varepsilon(q_x)/\delta q_x$ and shifts the wave packet position. The overlap

$$\left\langle P^2 \right\rangle = P_0^2 \int d\xi \, R(\xi) \left[\int_{-\infty}^{\infty} \Psi(x) \Psi(x+\xi) \right]^2$$



For a Gaussian initial wave packet of width w

$$\langle P^2 \rangle = P_0^2 \left(1 + \frac{11\sigma^2 L}{9w^2}\right)^{-1/2}.$$

in case of **δ-correlated static bond disorder**,

$$\langle P^2 \rangle = P_0^2 \left(1 + \frac{11}{9}\frac{L\nu}{w^2} \ln \frac{v_{gr}}{\omega_{ir}}\right)^{-1/2}$$

in case of **δ-correlated $1/f$ noise** and

$$\langle P^2 \rangle = P_0^2 \left(1 + \frac{v_{gr}^2}{w^2} \langle \chi^2(t) \rangle\right)^{-1/2}, \quad \langle \chi^2(t) \rangle = \int \frac{d\omega}{2\pi} \langle \xi_\omega^2 \rangle \left(\frac{\sin(\omega t/2)}{\omega/2}\right)^2$$

for **uniform noise** with arbitrary statistic $\langle \xi_\omega^2 \rangle$.

Main results

1. Protocol for two-qubit quantum operation via edge modes.
2. Two methods for quantum information transmission between the edge and the external qubit.
3. Fidelity of quantum operations in disordered system.

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