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Bianchi-I cosmologies, magnetic fields and singularities

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Introduction and Motivations

- Almost all modern cosmology is based on the Friedmann-Lemaître spatially homogeneous and isotropic cosmological models.

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right).$$

- The study of spatially homogeneous, but anisotropic models which takes its origin in the work by Luigi Bianchi, represents a great interest from both mathematical and physical points of view.
- The simplest spatially homogeneous and anisotropic cosmological model is the Bianchi - I model.

$$ds^2 = dt^2 - (a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2).$$

- The first general exact anisotropic solution for this metric in empty space was found by Kasner in 1921.

- In 1963 Khalatnikov and Lifshitz have begun applying the Bianchi universes (and in particular Bianchi -I universe) for the study of the problem of the singularity.
- At the end of sixties Belinski, Khalatnikov and Lifshitz have discovered the phenomenon of the oscillatory approach to the cosmological singularity.
- Later it was understood that when the universe tends to the singularity its dynamics becomes chaotic.
- Exact solutions for the Bianchi - I universe filled with isotropic perfect fluid were also studied (Heckmann and Schucking, Jacobs).
- The dynamics of the Bianchi-I universe filled with a magnetic field is particularly interesting (Rosen,Doroshkevich,Shikin,Thorne,Jacobs).
- The interest to the solutions involving the magnetic fields is not purely academical. The existence of large-scale magnetic fields in our universe is an important and enigmatic phenomenon.

- The goal of our work was to describe in detail the dynamics of the Bianchi-I universe in the presence of the magnetic field and to analyze it from the point of view of the new approach to the description of the singularity crossing.

Bianchi-I universes with a magnetic field

The Lagrangian of the electromagnetic field is

$$L_{\rm em} = -\frac{1}{16\pi} F_{ik} F^{ik}.$$

The energy-momentum tensor of the electromagnetic field is

$$T^i_k = \frac{1}{4\pi}\left(-F^{il}F_{kl} + \frac{1}{4}\delta^i_k F_{lm}F^{lm}\right).$$

If the electric field is absent and the magnetic field has the only component oriented along the third axes z, then the only non-zero component of the electromagnetic field tensor F_{ij} is F_{12}.

One of the Maxwell equations is

$$F_{[ij;k]} = 0.$$

In the spacetime without torsion

$$F_{[ij;k]} = 0.$$

Choosing the triplet of the indices $0, 1, 2$ we see that

$$F_{12,0} = 0.$$

That means the the only nonzero component of the two times covariant electromagnetic field tensor is constant.
The only nonvanishing component of the two times contravariant electromagnetic field tensor is

$$F^{12} = g^{11} g^{22} F_{12} \sim \frac{1}{a^2 b^2}.$$

The energy-momentum tensor components are:

$$T_0^0 = \frac{B_0^2}{a^2 b^2}, \;\; T_1^1 = T_2^2 = -\frac{B_0^2}{a^2 b^2}, \;\; T_3^3 = \frac{B_0^2}{a^2 b^2},$$

where B_0^2 is a positive constant, characterizing the intensity of the magnetic field.

It is convenient to use the following parametrization of three scale factors:

$$a(t) = R(t)e^{\alpha(t)+\beta(t)},$$
$$b(t) = R(t)e^{\alpha(t)-\beta(t)},$$
$$c(t) = R(t)e^{-2\alpha(t)}.$$

Then the components of the Ricci tensor are

$$R_0^0 = -\left(3\frac{\ddot{R}}{R} + 6\dot{\alpha}^2 + 2\dot{\beta}^2\right),$$

$$R_1^1 = -\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}(\dot{\alpha} + \dot{\beta}) + \ddot{\alpha} + \ddot{\beta}\right),$$

$$R_2^2 = -\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}(\dot{\alpha} - \dot{\beta}) + \ddot{\alpha} - \ddot{\beta}\right),$$

$$R_3^3 = -\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} - 6\frac{\dot{R}}{R}\dot{\alpha} - 2\ddot{\alpha}\right).$$

The scalar curvature is

$$\mathcal{R} = -\left(\frac{6\ddot{R}}{R} + 6\frac{\dot{R}^2}{R^2} + 6\dot{\alpha}^2 + 2\dot{\beta}^2\right).$$

The Einstein equations are:

$$R_0^0 - \frac{1}{2}\mathcal{R} = \frac{B_0^2 e^{-4\alpha}}{R^4},$$

$$R_1^1 - \frac{1}{2}\mathcal{R} = -\frac{B_0^2 e^{-4\alpha}}{R^4},$$
$$R_2^2 - \frac{1}{2}\mathcal{R} = -\frac{B_0^2 e^{-4\alpha}}{R^4},$$
$$R_3^3 - \frac{1}{2}\mathcal{R} = \frac{B_0^2 e^{-4\alpha}}{R^4}.$$

Note that in our case the scalar curvature should be equal to zero.

$$R^1_1 - R^2_2 = 2\ddot{\beta} + 6\frac{\dot{R}}{R}\dot{\beta} = 0$$

and, hence,

$$\dot{\beta} = \frac{\beta_0}{R^3},$$

just like in the Kasner and in the Heckmann-Schucking solutions.

$$R^1_1 + R^2_2 - 2R^3_3 = -6\ddot{\alpha} - 18\frac{\dot{R}}{R}\dot{\alpha} = -4\frac{B_0^2}{R^4}e^{-4\alpha},$$

or

$$\ddot{\alpha} + 3\frac{\dot{R}}{R}\dot{\alpha} = \frac{2B_0^2}{3R^4}e^{-4\alpha}.$$

We can construct also another combination of equations, arriving to

$$R^1_1 + R^2_2 + 2R^3_3 = -4\frac{\ddot{R}}{R} - 8\frac{\dot{R}^2}{R^2} + 6\frac{\dot{R}}{R}\dot{\alpha} + 2\ddot{\alpha} = 0$$

or

$$\ddot{\alpha} + 3\frac{\dot{R}}{R}\dot{\alpha} = 2\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2}\right).$$

The last equation can be rewritten as follows:

$$\frac{1}{R^3}\frac{d}{dt}(\dot{\alpha}R^3) = \frac{1}{R^3}\frac{d}{dt}(2\dot{R}R^2)$$

or

$$\frac{d}{dt}(\dot{\alpha}R^3) = \frac{d}{dt}(2\dot{R}R^2).$$

$$\dot{\alpha} R^3 = 2\dot{R}R^2 + \alpha_0$$

and

$$\dot{\alpha} = \frac{2\dot{R}}{R} + \frac{\alpha_0}{R^3}.$$

Combining equations, we obtain

$$\frac{B_0^2}{R^4} e^{-4\alpha} = \frac{1}{R^3} \frac{d^2 R^3}{dt^2}.$$

Knowing the expression for R we can find the expression for α. The second time derivative of the R^3 should be always positive.

Combining the preceding equations, we obtain the following equation for R:

$$9\frac{\dot{R}^2}{R^2} + 12\frac{\alpha_0}{R^3}\frac{\dot{R}}{R} + 3\frac{\alpha_0}{R^6} + \frac{1}{R^3}\frac{d^2R^3}{dt^2} = 0.$$

Introducing a new volume variable

$$V \equiv R^3,$$

we rewrite the preceding equation:

$$\frac{\dot{V}^2}{V^2} + 4\frac{\alpha_0}{V}\frac{\dot{V}}{V} + \frac{3\alpha_0^2}{V^2} + \frac{\beta_0^2}{V^2} + \frac{\ddot{V}}{V} = 0.$$

or

$$V\ddot{V} + \dot{V}^2 + 4\alpha_0\dot{V} + 3\alpha_0^2 + \beta_0^2 = 0.$$

Not all the solutions of this equations are the solutions of the complete system of Einstein and Maxwell equations. The second derivative of the variable V should be positive. Then

$$\ddot{V} = -\frac{1}{V}(\dot{V}^2 + 4\alpha_0\dot{V} + 3\alpha_0^2 + \beta_0^2).$$

The variable V should be always nonnegative, thus, the positivity of $\ddot{V}$ implies the following bounds on the value of the time derivative of V:

$$-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2} \leq \dot{V} \leq -2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}.$$

It is possible only if

$$\alpha_0^2 \geq \beta_0^2.$$

If α_0 is positive then $\dot{V}$ is negative and vice versa.

Let us consider the case when $\alpha_0 \geq 0$, which corresponds to the contracting universe.
The absolute value of $\dot{V}$ is decreasing satisfying always the constraint

$$|\dot{V}| \geq 2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2} = V_1.$$

We can consider two time moments t_1 and t_2 such that

$$V(t_1) = 0$$

and

$$\dot{V}(t_2) = V_1.$$

The qualitative analysis of the corresponding differential equation shows that these moments coincide:

$$t_2 = t_1.$$

In the vicinity of the singularity

$$V = |V_1|(t_1 - t) + B_1(t_1 - t)^{\mu},$$

$$\mu = 1 + \frac{2\sqrt{\alpha_0^2 - \beta_0^2}}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}.$$

One can see that

$$1 < \mu \leq 3.$$

We can find the expressions for the anisotropy factors α and β.

$$\beta = \int dt \frac{\beta_0}{R^3} = -\frac{\beta_0}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}} \ln(t_1 - t),$$

$$\alpha = \left(\frac{2}{3} - \frac{\alpha_0}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}\right) \ln(t_1 - t).$$

The expressions for the three scale factors:

$$a(t) \sim (t_1 - t)^{1 - \frac{\alpha_0 + \beta_0}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}},$$

$$b(t) \sim (t_1 - t)^{1 - \frac{\alpha_0 - \beta_0}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}},$$

$$c(t) \sim (t_1 - t)^{-1 - \frac{-2\alpha_0}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}}.$$

For definiteness, let us choose $\beta_0 \geq 0$.

We can rewrite these expressions in the Kasner form:

$$a(t) \sim (t_1 - t)^{p_1},$$
$$b(t) \sim (t_1 - t)^{p_2},$$
$$c(t) \sim (t_1 - t)^{p_3},$$

where

$$p_1 = \frac{\alpha_0 - \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2}}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}} < 0,$$
$$p_2 = \frac{\alpha_0 + \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2}}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}} > 0,$$
$$p_3 = \frac{\sqrt{\alpha_0^2 - \beta_0^2}}{2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}} > 0.$$

It is easy to check that the exponents p_1, p_2 and p_3 satisfy the Kasner relations:

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1.$$

That means that the presence of the magnetic field does not change the character of the singularity.

The contribution of the magnetic field into the Einstein equations:

$$\frac{B_0^2}{a^2 b^2} = \frac{\mu(\mu - 1)B_1}{A}(t_2 - t)^{\mu - 3}.$$

Here, $\mu - 3 > -2$ and hence this term is weaker than the terms $1/t^2$ arising due to the anisotropy and cannot influence the structure of the singularity.

Expanding universe, $\alpha_0 < 0, \dot{V} > 0$.
This expansion will be infinite and at $t \to \infty$ the time derivative $\dot{V}$ will tend to the critical value

$$V_2 = -2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}.$$

The behavior of $V(t)$ in the limit $t \to \infty$ is

$$V(t) = V_2 t - B_2 t^\nu,$$

$$\nu = 1 - \frac{2\sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}}.$$

$$\frac{1}{3} \leq \nu < 1.$$

$$a \sim t^{p_1},$$
$$b \sim t^{p_2},$$
$$c \sim t^{p_3},$$

$$p_1 = \frac{-\alpha_0 + \beta_0 + \sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}},$$
$$p_2 = \frac{-\alpha_0 - \beta_0 + \sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}},$$
$$p_3 = \frac{-\sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2}}.$$

The exponents satisfy the Kasner relations. That means that the presence of the magnetic field does not influence the structure of the metric at $t \to \infty$ and does not imply the isotropization of the universe, in contrast to the dust-like matter in the Heckmann-Schucking solution.

The energy density of the magnetic field at $t \to \infty$ is

$$\frac{B^2}{a^2 b^2} = \frac{\nu(\nu-1)B_2}{V_2} t^{\nu-3},$$

where $\nu - 3 < -2$ and is weaker than the anisotropy term.

What happens in the distant past, close to the initial singularity?

$$p_1' = \frac{-\alpha_0 + \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}},$$

$$p_2' = \frac{-\alpha_0 - \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}},$$

$$p_3' = \frac{\sqrt{\alpha_0^2 - \beta_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2}}.$$

To establish the relation between the set of the Kasner indices at the beginning and at the end of the evolution, it is convenient to use the Lifshitz-Khalatnikov parametrization. If

$$p_1 \leq p_2 \leq p_3,$$

then they can represented by means of a real parameter $u \geq 1$:

$$p_1 = -\frac{u}{1+u+u^2},$$
$$p_2 = \frac{1+u}{1+u+u^2},$$
$$p_3 = \frac{u(1+u)}{1+u+u^2}.$$

If we choose the following anisotropy parameters:

$$\alpha_0 < 0,\ \ \beta_0 < 0,\ \ |\beta_0| < \frac{3}{5}|\alpha_0|,$$

then

$$u = \frac{1 + u'}{u'} < 2.$$

Inversely,

$$u' = \frac{1}{u - 1}.$$

The evolution towards singularity includes two transformations. During the first transformation the parameter u' undergoes the shift $u \to u - 1$ and the roles of the axes, characterized by the exponents p_1 and p_3, are exchanged.
This transformation is called "change of the Kasner epoch" by BKL. As a result of this transformation we arrive to the value of the parameter $u - 1$ which is less than 1.
The next transformation called "change of Kasner era" makes $u - 1 \to \frac{1}{u-1}$.
This transformation exchanges the roles of the axes 2 and 3.

When the relation between the parameters α_0 and β_0 is opposite:

$$\alpha_0 < 0, \ \beta_0 < 0, \ 1 > |\beta_0| > \frac{3}{5}|\alpha_0|,$$

we have

$$u = u' + 1,$$

or, inversely,

$$u' = u - 1,$$

and we have only a change of the Kasner epoch.

Remarkably, the transition between two Kasner regimes in the model with magnetic field is characterized by the same law as in the case of an empty Bianchi-II universe.

In the case of Bianchi-IX or Bianchi-VIII models the universe passes through an infinite series of the changes of the Kasner epochs and eras.

The exact differential equation for the volume can be rewitten as follows:

$$\frac{d}{dt}\left(\dot{V}V + 4\alpha_0 V\right) = -(3\alpha_0^2 + \beta_0^2),$$

$$\dot{V}V + 4\alpha_0 V = -(3\alpha_0^2 + \beta_0^2)t.$$

Let us introduce a new variable X

$$V = Xt.$$

$$t\dot{X} + X + 4\alpha_0 + \frac{3\alpha_0^2 + \beta_0^2}{X} = 0,$$

$$\frac{dX}{X + 4\alpha_0 + \frac{3\alpha_0^2+\beta_0^2}{X}} = -\frac{dt}{t}.$$

$$
\begin{aligned}
&\frac{1}{2}\ln\left(1+\frac{4\alpha_0 X + X^2}{3\alpha_0^2+\beta_0^2}\right)\\
&+\frac{\alpha_0}{\sqrt{\alpha_0^2-\beta_0^2}}\left(\ln\left(1+\frac{X}{2\alpha_0+\sqrt{\alpha_0^2-\beta_0^2}}\right)\right.\\
&\left.-\ln\left(1+\frac{X}{2\alpha_0-\sqrt{\alpha_0^2-\beta_0^2}}\right)\right)=\ln\frac{t_0}{t}.
\end{aligned}
$$

We cannot invert this expression and find X as a function of t. Thus, we cannot use this equation to find exact expressions for the anisotropy factors.

Bianchi-I universes with a magnetic field and a perfect fluid I. Dust

$$\rho = \frac{\rho_0}{abc} = \frac{\rho_0}{R^3} = \frac{\rho_0}{V},$$

$$\mathcal{R} = -\rho.$$

$$\frac{d(\dot{\alpha} R^3)}{dt} = \frac{2}{3}\frac{d^2 R^3}{dt^2} - \rho_0.$$

$$\dot{\alpha} = 2\frac{\dot{R}}{R} + \frac{\alpha_0}{R^3} - \frac{\rho_0 t}{R^3}.$$

$$\frac{B_0^2}{R^4}e^{-4\alpha} = \frac{1}{R^3}\frac{d^2R^3}{dt^2} - \frac{\rho_0}{R^3},$$

which implies

$$\frac{d^2R^3}{dt^2} > \rho_0.$$

$$X\ddot{V} + \dot{V}^2 + 4\alpha_0\dot{V} + (3\alpha_0^2 + \beta_0^2)$$
$$-4\rho_0\dot{V}t - 6\alpha_0\rho_0 t + 3\rho_0^2 t^2 = 0.$$

We are not able to find the solution of this equation.

When the universe is close to the singularity, the influence of terms, proportional to ρ_0 is negligible and we have a Kasner type singularity with a positive Kasner exponent corresponding to the axis along which the magnetic field is directed.

When the volume of the universe grows and tends to the infinity, we encounter an opposite situation. The term $3\rho_0 t^2$ dominates, $V \sim t^2$ and we have the isotropization just like in the standard Heckmann-Schucking solution.

II. Massless scalar field - stiff matter

The Klein-Gordon equation for the massless scalar field in the Bianchi-I universe has the form

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} = 0.$$

Its solution is

$$\dot{\phi} = \frac{\tilde{\phi}_0}{R^3} = \frac{\tilde{\phi}_0}{V}.$$

$$T_0^0 = \frac{\phi_0^2}{V^2},$$

$$T_1^1 = T_2^2 = T_3^3 = -\frac{\phi_0^2}{V^2}.$$

That means that in all the formulas for the empty space we should substitute $\beta_0^2 \to \beta_0^2 + \phi_0^2$.

$$a(t) \sim t^{p_1},$$
$$b(t) \sim t^{p_2},$$
$$c(t) \sim t^{p_3},$$

$$p_1 = \frac{-\alpha_0 + \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}},$$
$$p_2 = \frac{-\alpha_0 - \beta_0 - \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}},$$
$$p_3 = \frac{\sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 - \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}.$$

The Kasner relations are changed:

$$p_1 + p_2 + p_3 = 1,$$

$$p_1^2 + p_2^2 + p_3^2 = 1 - q^2,$$

$$q^2 = \frac{2\phi_0^2}{4\alpha_0^2 + 4\alpha_0\sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2} + \alpha_0^2 - \beta_0^2 - \phi_0^2}.$$

$$0 \leq q^2 < \frac{1}{2}.$$

In the case of the universe filled only with the massless scalar field the bound on the parameter q^2 is less stringent:

$$0 \leq q^2 \leq \frac{2}{3}.$$

The presence of the parameter q^2 indicates some kind of the isotropization of the universe.
The limiting value $q^2 = \frac{2}{3}$ means that $p_1 = p_2 = p_3 = \frac{1}{3}$, i.e. that the expansion of the universe is totally isotropic.
The presence of the magnetic field makes such a high degree of the isotropization impossible.

The expressions for the Kasner exponents of the universe when its volume tends to infinity:

$$p_1' = \frac{-\alpha_0 + \beta_0 + \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}},$$

$$p_2' = \frac{-\alpha_0 - \beta_0 + \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}},$$

$$p_3' = \frac{-\sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}{-2\alpha_0 + \sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2}}.$$

$$(p_1')^2 + (p_2')^2 + (p_3')^2 = 1 - q'^2,$$

$$q'^2 = \frac{2\phi_0^2}{4\alpha_0^2 - 4\alpha_0\sqrt{\alpha_0^2 - \beta_0^2 - \phi_0^2} + \alpha_0^2 - \beta_0^2 - \phi_0^2} < q^2.$$

When we go from the singularity towards an infinite expansion the value of the parameter q^2 decreases and hence, the level of the anisotropy increases in contrast to the isotropization induced by the presence of dust in the Heckmann-Schucking solution.

How to cross the singularity?

Recently some approaches to the problem of the description of the singularity crossing were elaborated. Behind these approaches there are basically two general ideas.
Firstly, to cross the singularity one must give a prescription matching non-singular, finite quantities before and after such a crossing.
Secondly, such a description can be achieved by using a convenient choice of field parametrization.

For our model we have invented two descriptions which give the same result.

Scenario 1 - negative volume

Let us tackle the problem studying the behaviour of the differential equations “behind the singularity”, where these equations and their solutions are more or less well defined while the variable can be come detached from the their initial origin. For example, the variable defined as a volume, can become negative.

Let us come back to the equation

$$\ddot{V} = -\frac{1}{V}(\dot{V}^2 + 4\alpha_0\dot{V} + 3\alpha_0^2 + \beta_0^2).$$

with $\alpha_0 > 0$, describing the contraction of the universe. Then we can consider the evolution of the universe in the region of the phase space, where the volume function V is negative.

After the period of "negative expansion" it arrives to a minimal value and then the period of "negative contraction" begins. At the end the universe crosses the singularity once again, re-entering the "normal" region with the same characteristics which it had crossing the singularity for the first time. Somewhere on the road the sign of the constant α_0 should be changed.

Scenario 2 - negative scale factors

All the Einstein equations for our system are invariant with respect to the change of the signs of the scale factors.
In these equations we encounter only the terms which look like $\frac{\ddot{a}}{a}$, $\frac{\dot{a}}{a}$ or a^2.
We can change the signs of all three scale factors, in this case the sign of the volume changes and its becomes again positive.
To make the equation for the volume variable V invariant with respect to the change of the sign of the volume , we should change also the sign of the parameter α_0.
The positive time derivative of the volume $\dot{V}$ immediately acquires a lower critical value $2|\alpha_0| - \sqrt{\alpha_0^2 - \beta_0^2}$ and an infinite expansion begins.

Conclusions

- Studying Bianchi - I spacetimes one can discover a lot of interesting phenomena.
- An interesting problem: is it possible to construct a Bianchi-I universe with a magnetic field which is not oriented along of the main axes?