Helical crystals: band structure, multicritical behavior and topological defects

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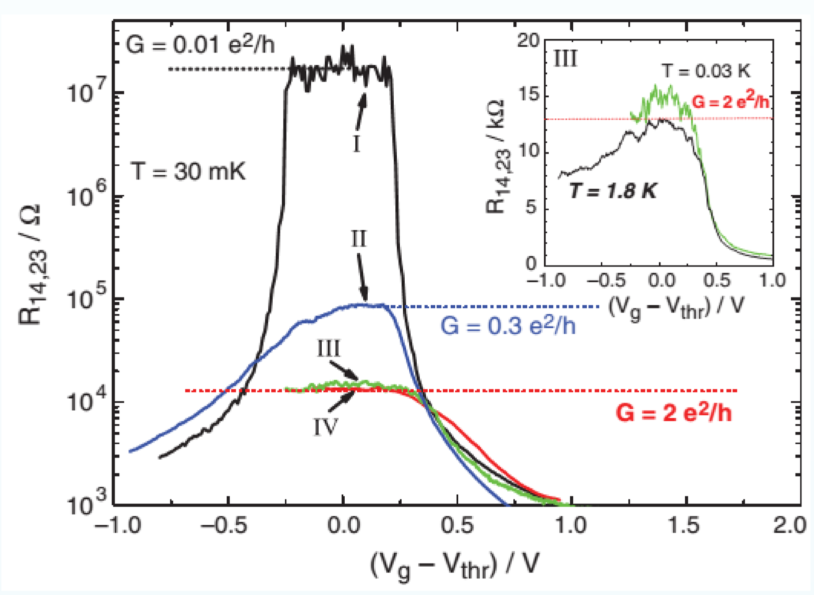
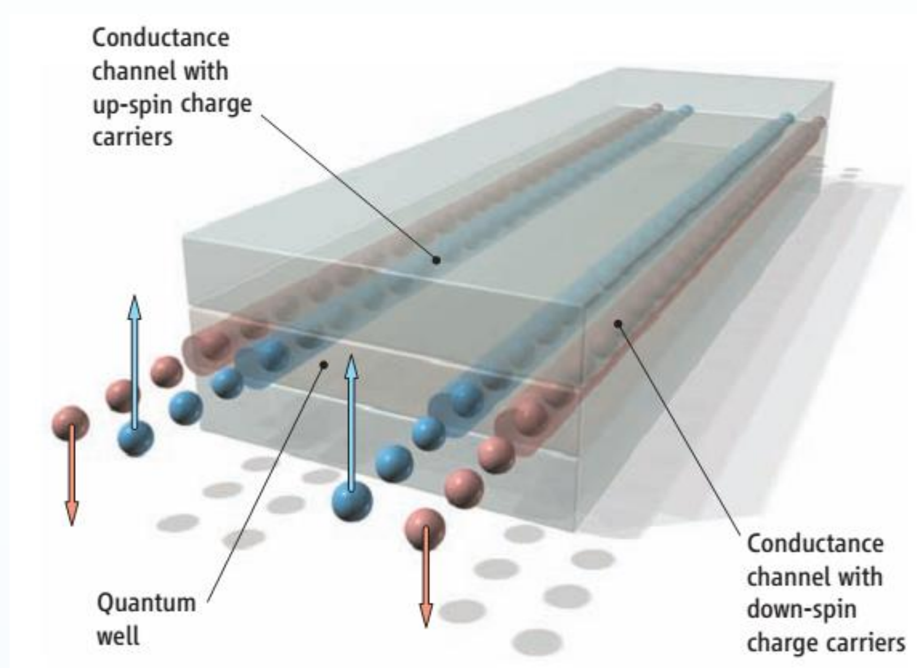
Helical states

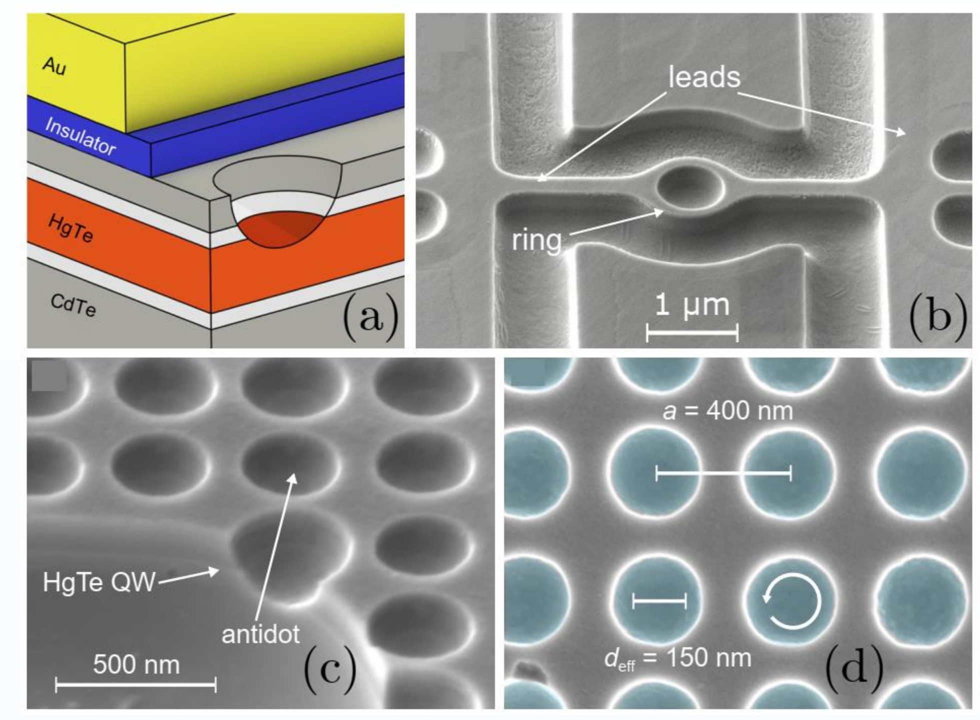
- Exist at the edge of 2D topological insulators
- Electrons with different spins propagate in opposite directions
- Protected by time reversal inversion





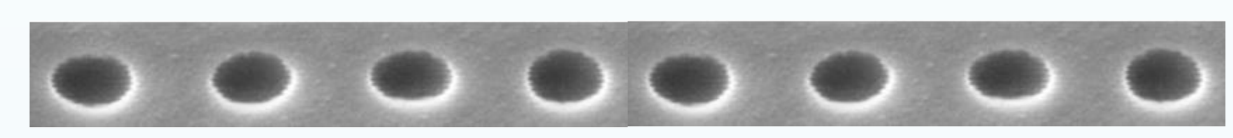
Topological insulator: quantum well based on HgTe/(Hg,Cd)Te [*König et. al. Science (2007)*]

Motivation

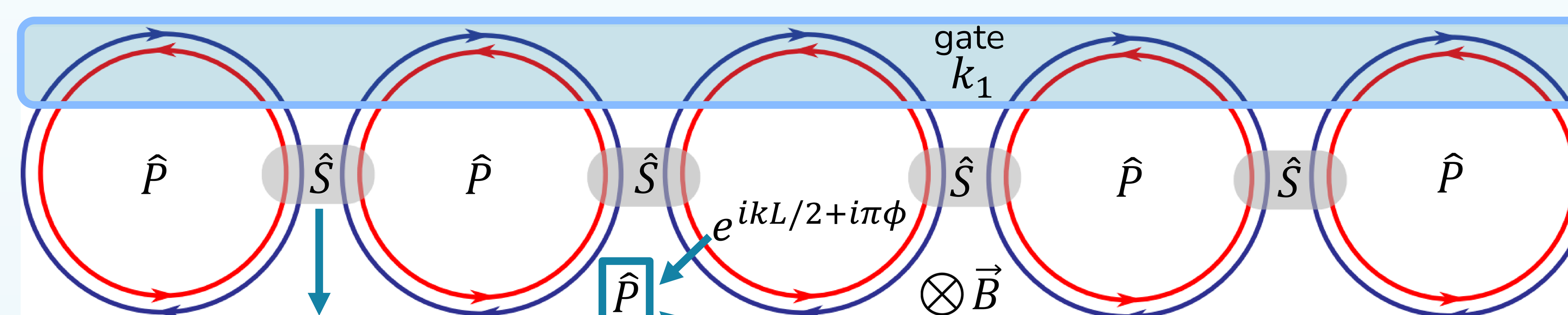


Array of antidots on the surface of 2D TI

1. *Maier, H.; Ziegler, J.; Fischer, R.; Kozlov, D.; Kvon, Z. D.; Mikhailov, N.; Dvoretsky, S. A. & Weiss, D. Ballistic geometric resistance resonances in a single surface of a topological insulator Nat. Commun. (2017)*
2. *PhD thesis Ziegler, J. (2018). Quantum transport in HgTe topological insulator nanostructures.*



1D helical crystal



Unitary and T-invariant

$$\hat{S} = \begin{pmatrix} 0 & t & f & r \\ t & 0 & -r^* & f^* \\ -f & -r^* & 0 & t^* \\ r & -f^* & t^* & 0 \end{pmatrix}$$

T_0

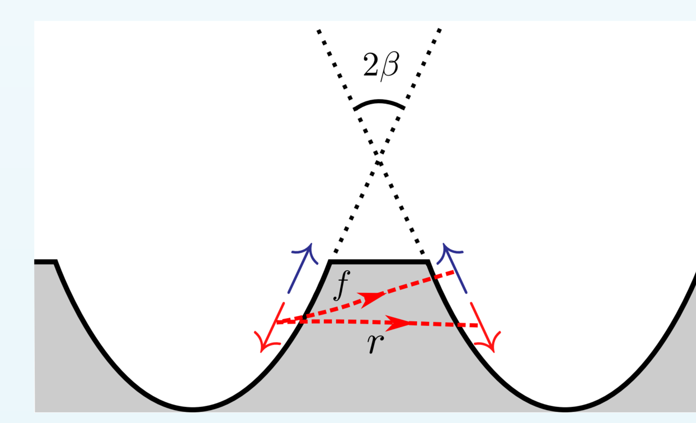
$t = \cos\alpha$, $r = e^{i\gamma}\sin\alpha\cos\beta$, $f = \sin\alpha\sin\beta$

L – ring length
ϕ – magnetic flux

Dynamic phase difference is added to the phase γ

$\gamma_{\text{eff}} \to \gamma + k_1 L_1 - k_2 L_2$

α – tunneling between rings
$\gamma = \pi/2$ symmetrical case



β – angle between spin quantization axes in neighboring edge states

$T = \hat{P} T_0$ transfer matrix passing the entire ring

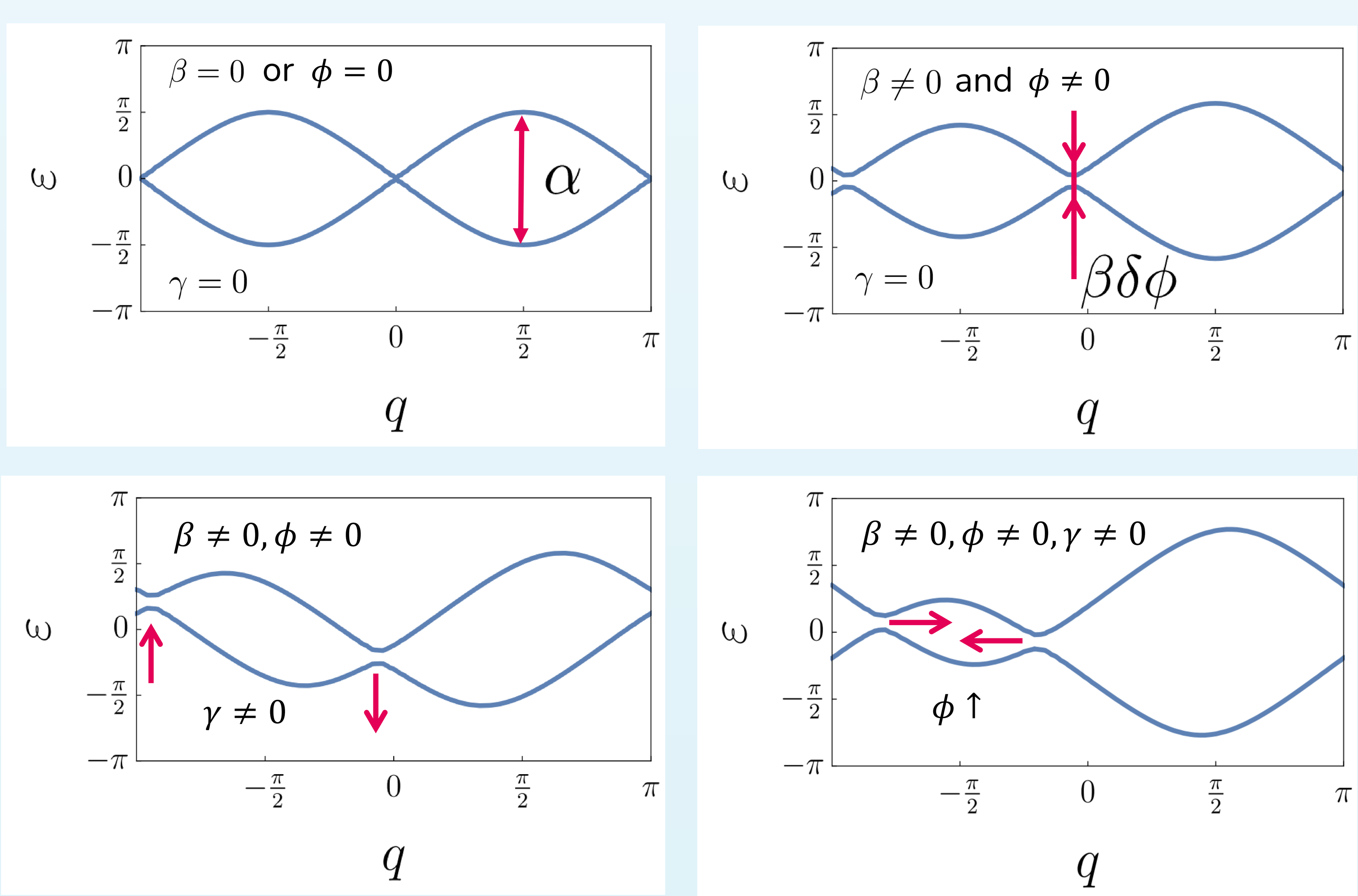
Crystal dispersion

$$\det[e^{iq} - T(kL = \varepsilon)] = 0$$

$\varepsilon(q) - ?$ ε – dimensionless energy, q – quasi-momentum

$$\left(\cos\beta\sin\left(\frac{\varepsilon}{2}+\pi\phi\right)+\sin\alpha\sin(\gamma+q)\right)\left(\cos\beta\sin\left(\frac{\varepsilon}{2}-\pi\phi\right)-\sin\alpha\sin(q-\gamma)\right) + \frac{1}{2}\sin^2\beta\left(\sin^2\alpha\cos 2\gamma+\cos^2\alpha\cos 2\pi\phi-\cos\varepsilon\right)=0$$

There are special values of the magnetic flux $\phi = 0; 1/2$, at which Dirac points appear



Dispersion manipulation: changing magnetic field and gate voltage

Localized states

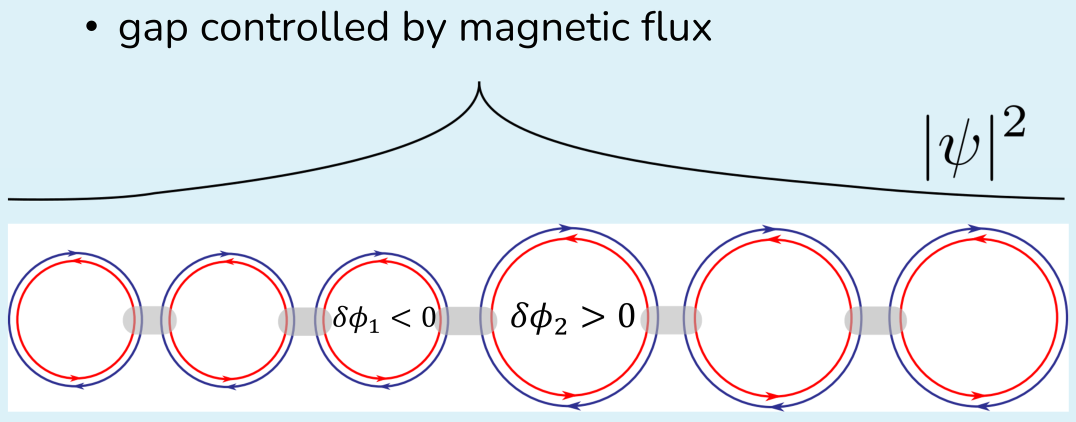
Dispersion near Dirac points

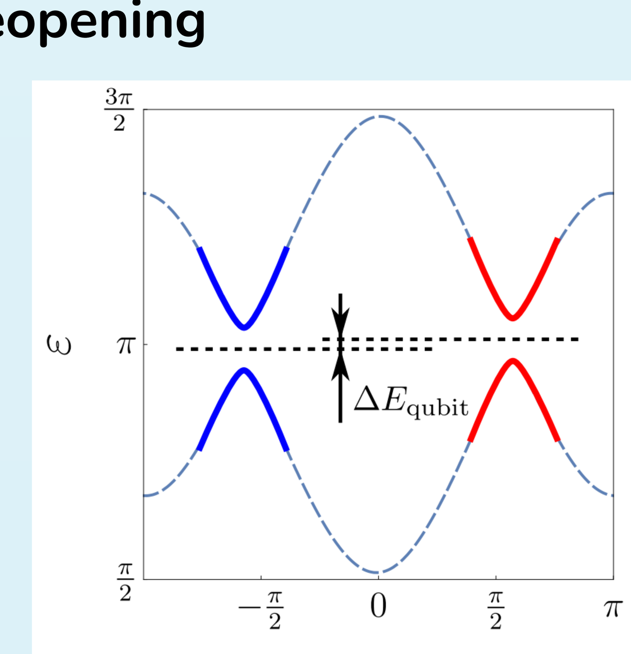
$$\frac{\delta\varepsilon^2}{4} = \delta q^2\alpha^2\cos^2\gamma + \frac{\beta^2\pi^2\delta\phi^2}{\cos^2\gamma}$$

Change sign of δϕ – gap reopening

Localized states of the Volkov-Pankratov type located inside the gap arise.

- splitting can be manipulated by gates and magnetic flux
- gap controlled by magnetic flux

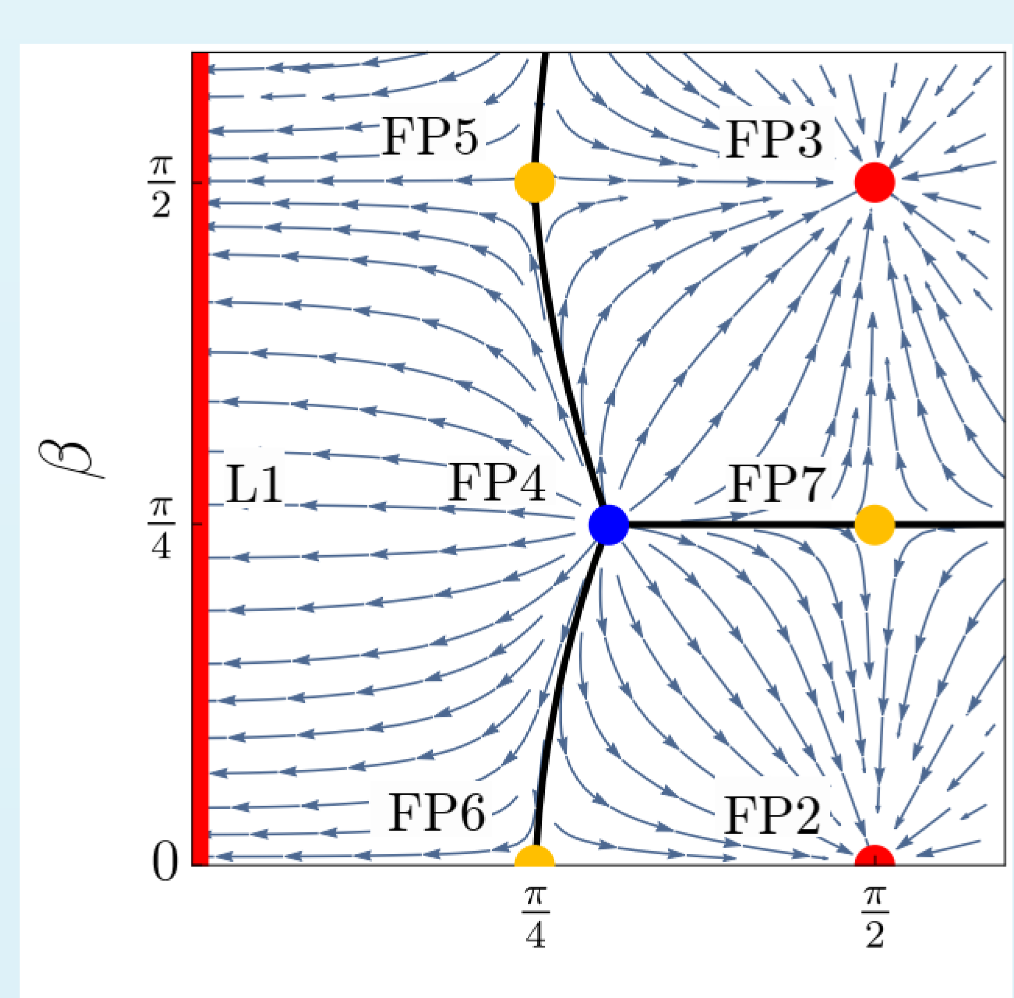




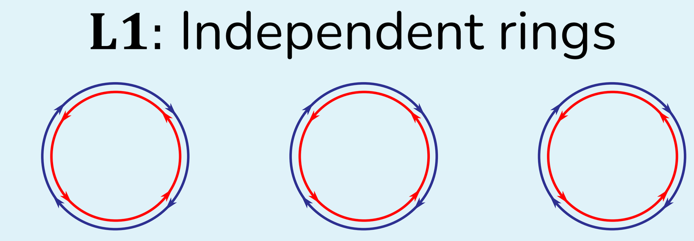
Electron interaction

It is known that the interaction renormalizes “the junction” between helical states. And what about a set of junctions? Renormalization - taking into account virtual processes $\lambda_F < l < l_T$
At high temperature $T \gg \frac{v_F}{L} \Leftrightarrow l_T \ll L$: junctions are renormalized separately
At low temperature $l_T \gg L$: on scale $l < L$ junctions are renormalized separately, further $L < l < l_T$ renormalization of the effectively homogeneous Luttinger fluid.

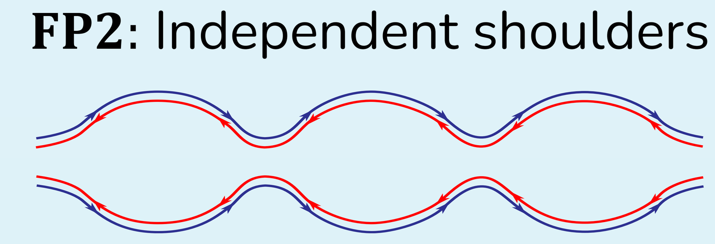
Result: renormalization of parameters α and β



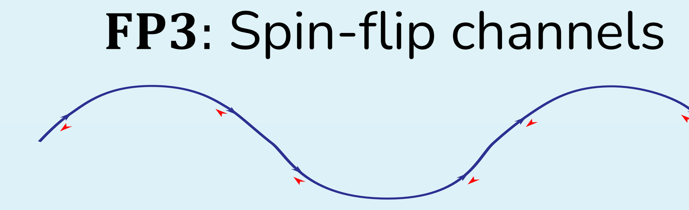
Attractive fixed points:
L1: Independent rings



FP2: Independent shoulders



FP3: Spin-flip channels



FP4: Multicritical point
Symmetric t = f = r = $1/\sqrt{3}$

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More details and references:
R. A. N., *D. N. Aristov, V. Yu. Kachorovskii, Tunable helical crystals, arXiv:2305.08242*