

# CHIRAL ANOMALY and MULTY-FLUID DYNAMICS of SLIDING CHARGE DENSITY WAVES.

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Charge Density Wave CDW in a quasi 1D electronic system.

Particle gap, amplitude and phase of the complex order parameter

Collective charges and currents,

topological objects: space vortices = dislocations,

space-time vortices = phase slips = real-time instantons.

Numerical modeling based on the TDGL : successes and warnings

**Consistent phenomenology with explicit account for fermions**

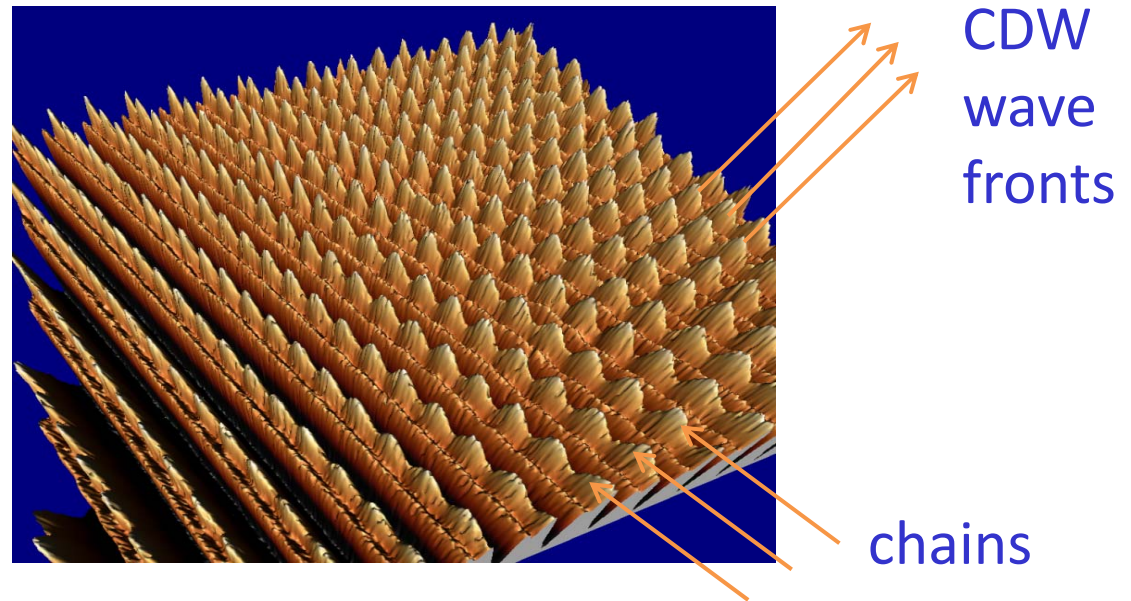
**Challenging numerical modeling for a non-analytical action**

Role of the chiral anomaly and its origin in

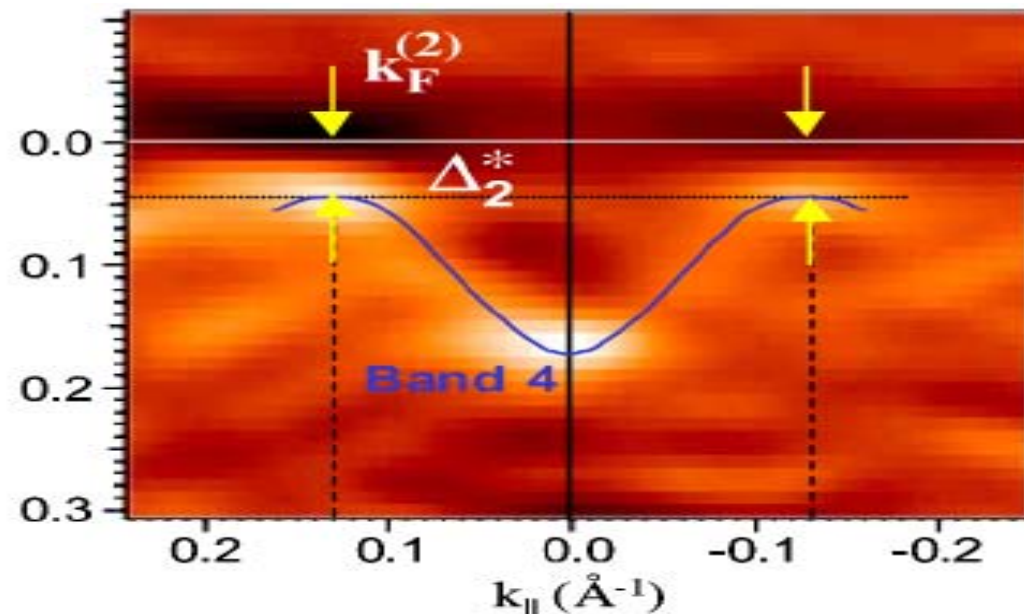
language of the solid state physics

# Incommensurate Charge Density Waves - CDW

Scanning Tunneling  
Microscopy shows  
the CDW modulation in  
atomic displacements  
and the electronic density.  
*Brun, Wang, Monceau, SB*



ARPES gives electrons'  
spectral density, hence  $E(k)$   
with the gap  $\Delta$   
formed by the CDW  
which wave number fits the  
Fermi diameter  $2k_F$



**CDW** =  $A \cos(2K_F x + \varphi)$   $\rightarrow$  complex order parameters  $\eta = A \exp[i\varphi]$

Degeneracy in the phase  $\varphi$  hence static vortices = dislocations,  
phase slips = instantons =  $(\mathbf{x}, \mathbf{t})$  vortices

Discrete degeneracy in sign of **A**.

### ***Collective effects related to the translational degeneracy breaking***

- Frohlich conduction by the collective sliding.
- Topological defects: solitons, dislocations (electronic vortices).
- Phase slips = real time instantons = spacio-temporary vortices
- Conversion among normal and condensed electrons.

At the nominal amplitude  $A=1$ :

the collective density  $\rho = \partial_x \varphi / \pi$  - CDW dilatation

the collective current  $j = -\partial_t \varphi / \pi$

The charge conservation law is satisfied **automatically** !

$$\frac{\partial}{\partial x} \partial_t \varphi + \frac{\partial}{\partial t} (-\partial_t \varphi) = 0$$

While the phase velocity or its deformation is the principle ingredient, no collective current can be set in without deformations of the CDW amplitude  $A(x,t)$ , particularly with  $A$  passing through zero within the vortex core or at the phase-slip event.

## Ginzburg-Landau (time dependent TDGL)– like model

$$\psi = A \exp(i\varphi)$$

$$H_{CDW} = \int d^3r \left\{ \left[ \left| \frac{\partial \psi}{\partial x} \right|^2 + \alpha \left| \frac{\partial \psi}{\partial y} \right|^2 \right] + |\psi|^2 \ln \frac{|\psi|^2}{e} \right\}$$
$$H_{\text{int}} = \int d^3r \left[ \Phi A^2 \partial_x \varphi / \pi + \Phi n + F(n) - |\nabla \Phi|^2 / 8\pi \right]$$

$\Phi$  - electric potential,

$n$  – concentration of normal carriers,  $F$  is their free energy

**Only extrinsic (external, from other bands) carriers  $n$  are taken explicitly. In the GL spirit, the intrinsic carriers (in the gap region) are integrated out, their effect is hidden in the CDW amplitude  $A=|\Psi|$ .**

Implementations:

Dissipative eqs. generated by variation over  $\Psi$

+ Poisson eq. for  $\Phi$  + diffusion eq for  $n$

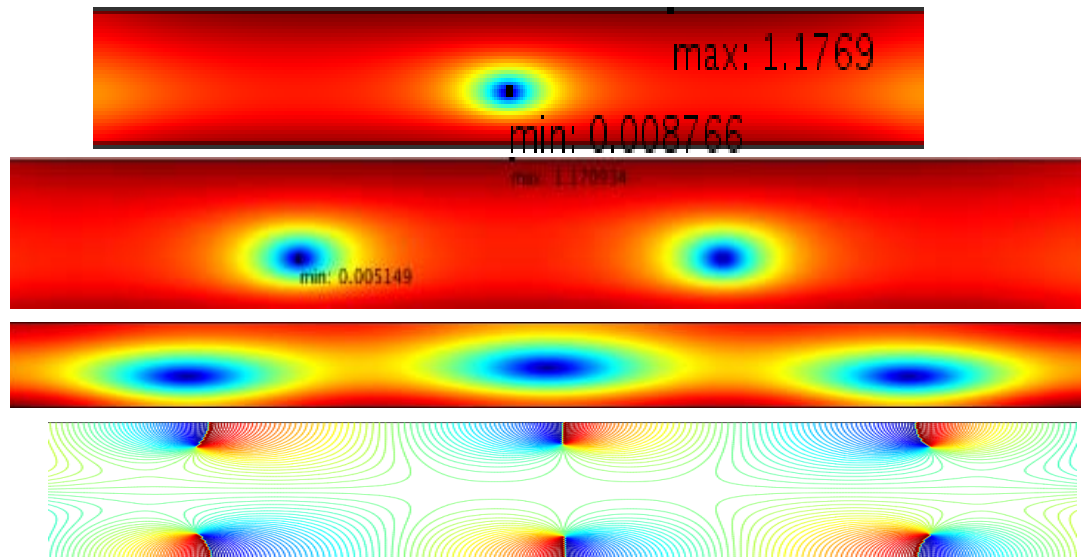
**Numerical solution of quite cumbersome PDE in 2+1 dimensions.**

**Additionally complicated by the long range Coulomb forces**

**via static Poisson eq.**

**Static vortex configurations for reconstruction of a CDW state  
under applied transverse electric field.  
Analogy to vortex state in superconductors under the magnetic field.**

Sequence of vortex enterings is registered as step-wise I-V in experiments

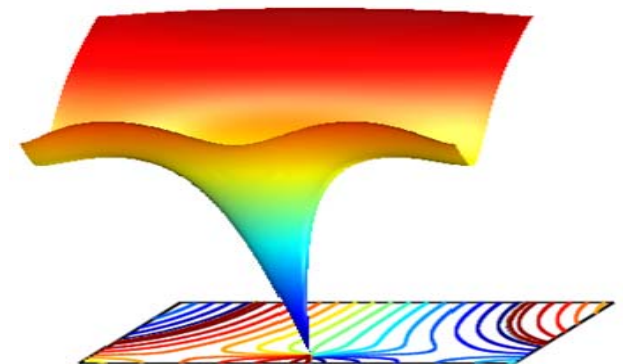


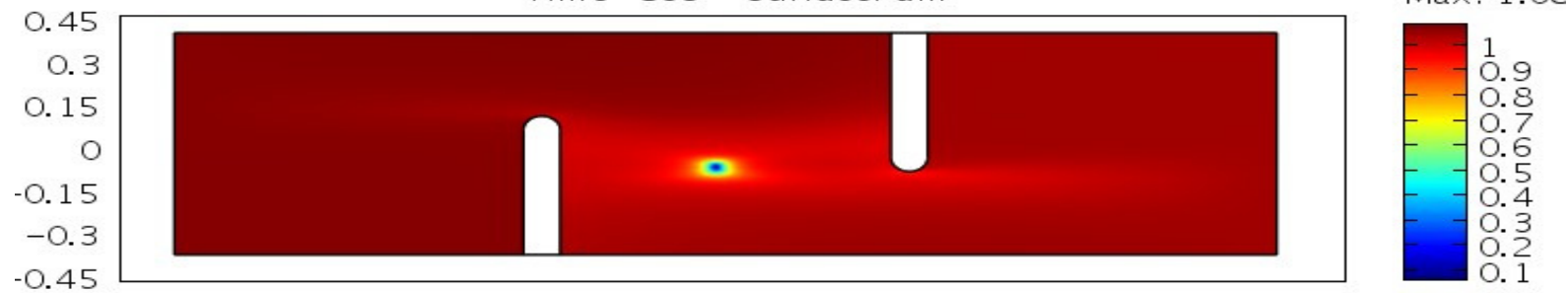
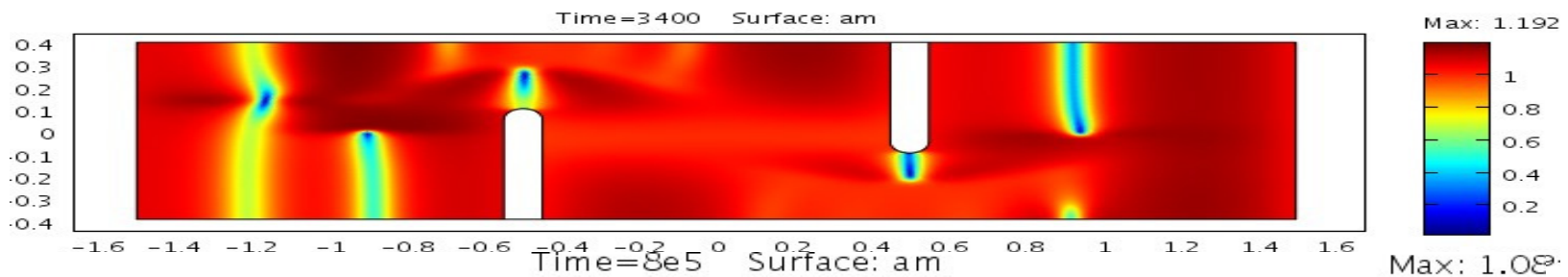
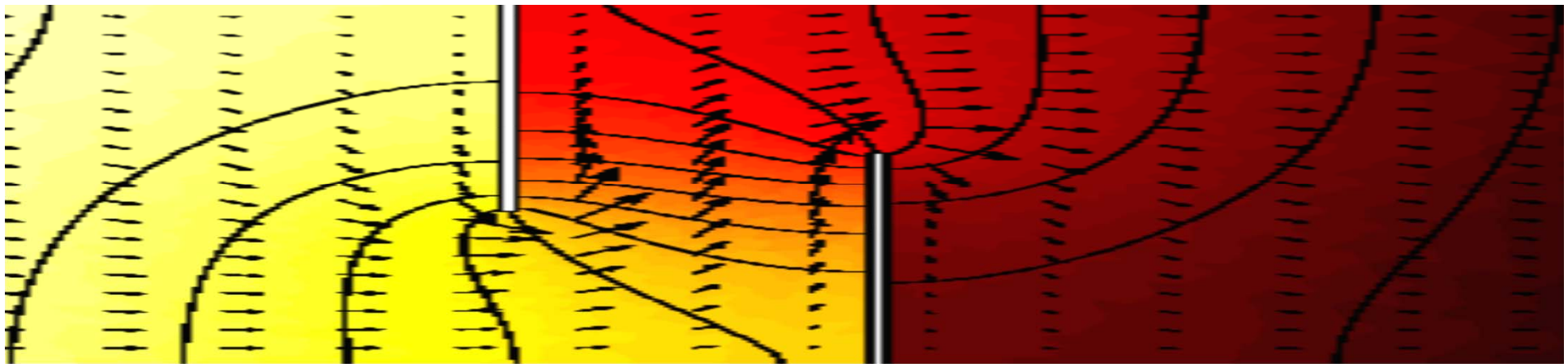
Amplitude **A**  
density plots



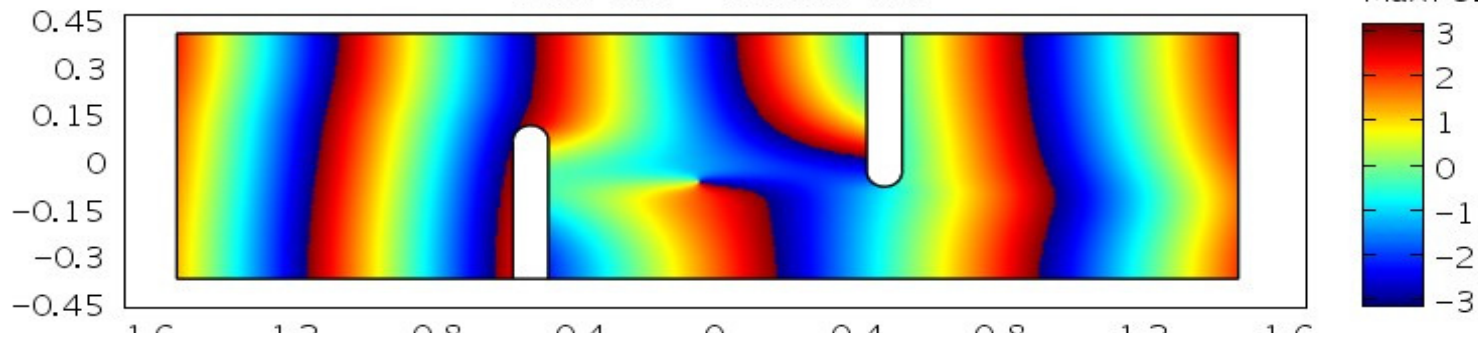
$2\pi$  phase rotations for a double array of vortices

Single vortex: 3D plot for **A**  
on top of line plot for phase





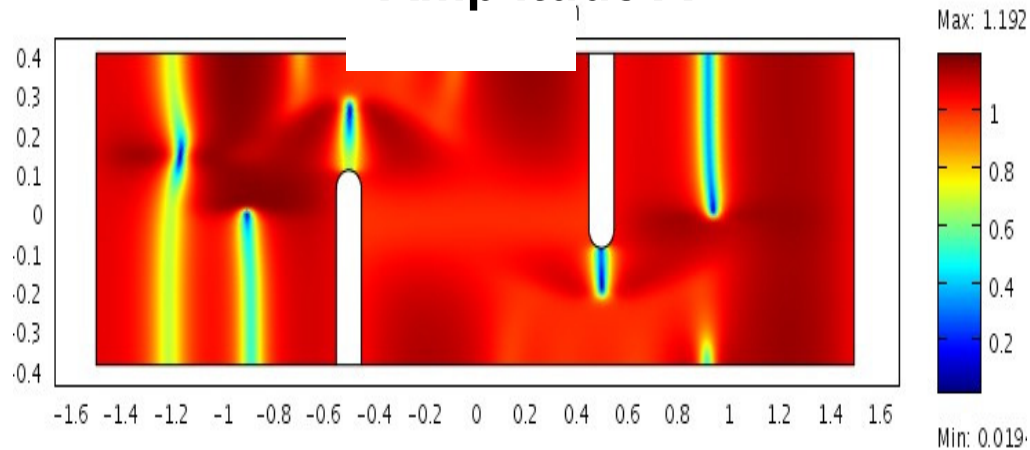
phase



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## Real geometry: initial short time fast dynamics

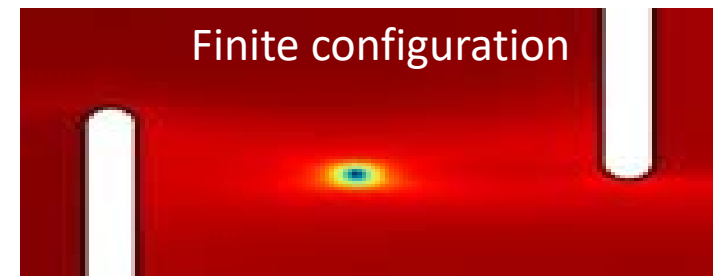
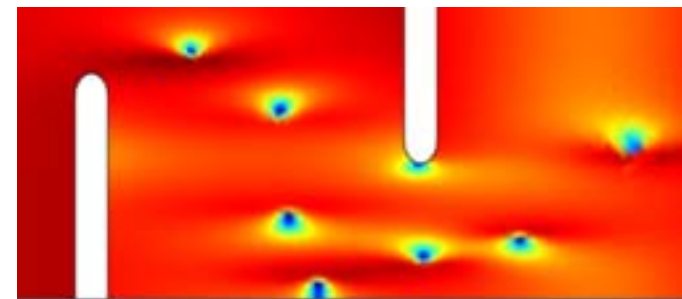
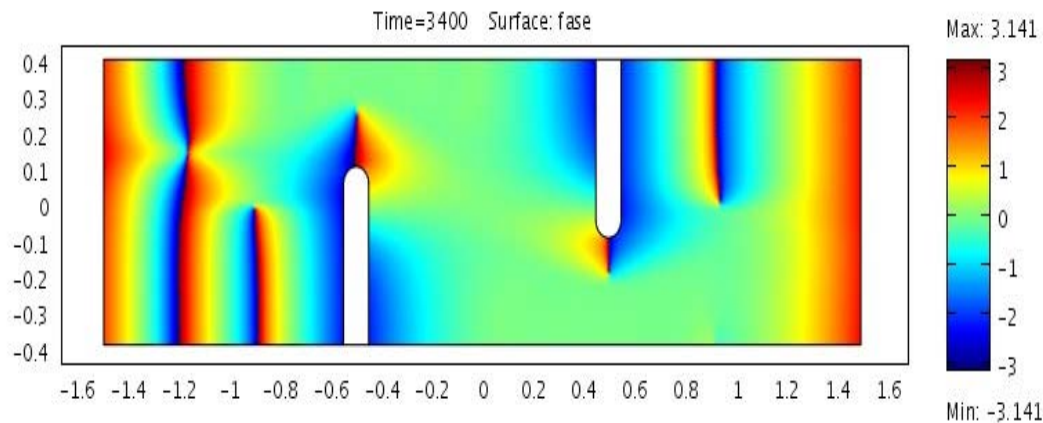
**Amplitude A**



All these 5 flashes are the phase-slip processes serving to redistribute the CDW collective charge

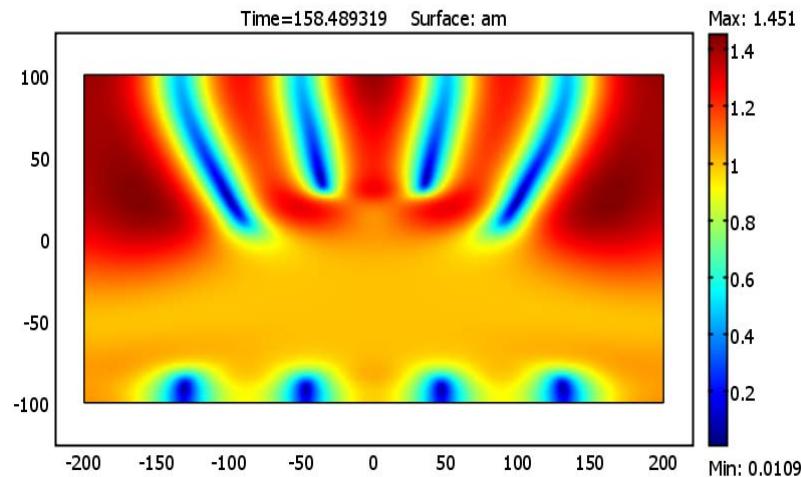
Intermediate configuration with many vortices originating both inside and outside of the junction.

**Phase** □



# Numerical modeling of nonstationary processes in CDWs within the TDGL phenomenology (+ electric field + free carriers)

*T. Yi, A. Rojo, N. Kirova and SB.*



traces of  $A=0$

nucleations

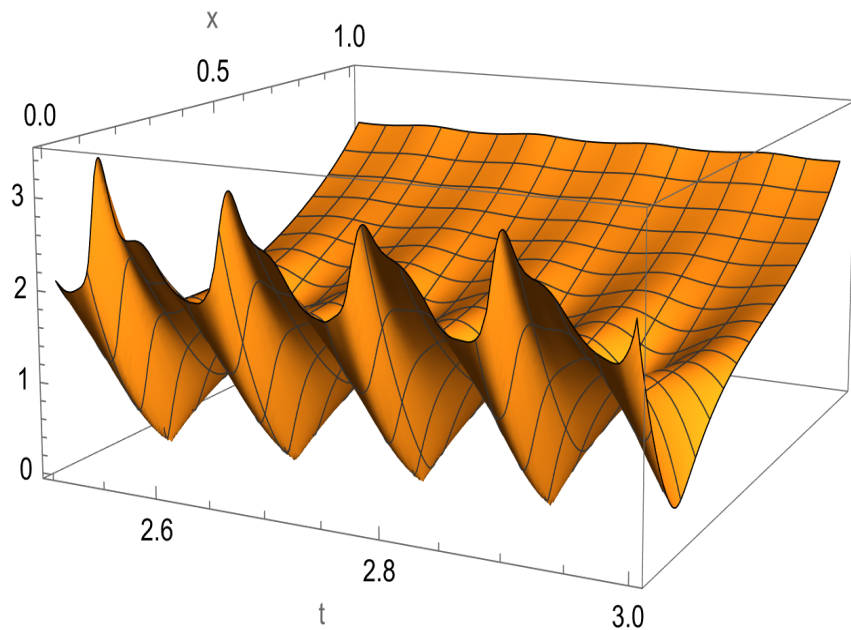


Rectangle.avi

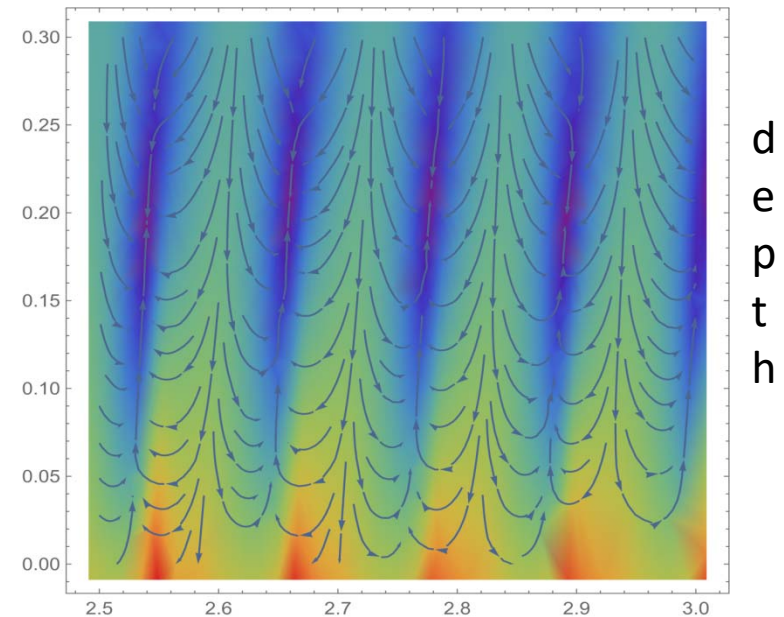
Many vortices appear temporarily in the course of the evolution. For that run, only one will be left.

## Phase slips = space-time vortices = real time instantons in $D=1+1$

This is how the current decays in a superconducting ring or normal fermions are converted to/from the condensate via the spectral flow through the gap in the course of the phase slip winding



Amplitude and vector of phase



time

## Hidden problems with the TDGL model for CDW

**Well established** and works for stationary state and as a tool to reach it.  
Takes explicitly the extrinsic carriers (not interacting with the CDW)

### Restrictions:

The intrinsic carriers have been integrated out and come into the model only via the order parameter amplitude  $A$  and related parameters.

### Major problem:

Violation of the local charge conservation for the condensate .

*automatically if  $A = \text{const}$*

$$n_c = \frac{A^2}{\pi} \frac{\partial \varphi}{\partial x} \quad j_c = -\frac{A^2}{\pi} \frac{\partial \varphi}{\partial t} \quad \longrightarrow \quad \frac{dn}{dt} = \frac{\partial n_c}{\partial t} + \frac{\partial j_c}{\partial x} = 0$$

In our case  
 $A(x,y,t) \neq \text{const}$

$$\pi \frac{dn}{dt} = \frac{\partial A^2}{\partial x} \frac{\partial \varphi}{\partial t} - \frac{\partial \varphi}{\partial x} \frac{\partial A^2}{\partial t} \neq 0 \quad \text{WRONG}$$

**Way of resolution:** keep normal carriers in hand and decompose  $n, j$

## More general scheme from the microscopics:

$\psi=(\psi_+,\psi_-)$  - electronic wave function components near  $\pm p_F$   
decomposed in right and left moving fermions

$\Delta e^{i\varphi}$  - order parameter

$\Phi$  and  $\mathbf{A}_x$  - scalar and vector potentials,

$v_F$  - Fermi velocity

$$H_{el} = \begin{pmatrix} i\frac{\partial}{\partial t} - i\hbar v_F \frac{\partial}{\partial x} + e\Phi + \frac{e}{c} A_x & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & i\frac{\partial}{\partial t} + i\hbar v_F \frac{\partial}{\partial x} + e\Phi - \frac{e}{c} A_x \end{pmatrix}$$

Not convenient: the gap  $\Delta$  is loaded with the essentially variable  $\mathbf{x}, \mathbf{t}$  dependent factor  $\exp(\pm i \varphi)$

Chiral transformation:  $\psi_{\pm} \rightarrow \psi_{\pm} e^{\pm i\phi/2}$

actually puts the electrons to the breathing frame of shifted

Fermi momentum and Fermi energy:  $\delta\mathbf{P}_F = \partial_x \phi / 2 \rightarrow \delta\mathbf{E}_F = \hbar \mathbf{v}_F / 2 \partial_x \phi$

Now the gap  $\Delta$  is unloaded from the phase factor,  
we arrive at a 1D semiconductor model,  
but in expense of elongating the applied potentials:

$$e\Phi \rightarrow e\Phi + \hbar v_F / 2 \partial_x \phi$$

$$e / cA_x \rightarrow e / cA_x + \hbar v_F / 2 \partial_t \phi$$

$$eE = -\partial_x \Phi + \partial_t A_x \rightarrow F = eE + (\partial_t^2 - \partial_x^2) \phi / 2$$

F – chiral invariant effective electric field

$\mathbf{F}$  – the chiral invariant effective electric field is felt by electrons.

Resulting energy, collective charge and current, etc.

might be functions of entire  $\mathbf{F}$  only.

E.g. the perturbed energy is expressed via the dielectric permittivity  $\epsilon$  of a semiconductor with gap  $\Delta$  at presense of remnant carriers:

$$W_e = -\frac{\epsilon(k, \omega) F^2}{8\pi} = -\frac{\epsilon(k, \omega)}{8\pi e^2} \left( -\frac{\partial e\Phi}{\partial x} - \frac{\hbar v_F}{2} \frac{\partial^2 \varphi}{\partial x^2} \right)^2$$

As correctly derived as wrong in result:

misses or distorts all expected contributions from the CDW phase – a semiconductor does not slide.

What was wrong?

Missed non-perturbative contribution known in field theory as the “chiral anomaly” (*rem. Krive for CDWs, Yakovenko for FISDWs*)

Action = free energy **W** after integration over fermions depends only of the complete chiral invariant potential:

$$W = W(e\Phi + \hbar v_F / 2 \partial_x \phi) \text{ can choose } e\Phi + \hbar v_F / 2 \partial_x \phi = 0$$

**False** local “gauge” invariance – potential  $\Phi$  can be excluded at no cost  
**Then no system reaction to application of  $\Phi$**

e.g. in absence of normal carriers we definitely expect:

$$W = \frac{\rho^2}{2 N_F} + e \rho \Phi \Rightarrow \frac{\hbar v_F}{4 \pi} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{e \Phi}{\pi} \frac{\partial \phi}{\partial x}$$

- Not a full square of the chiral-invariant combination:

$\sim \Phi^2$  looks to be an anomaly - and not only



With the linearized bottomless spectrum, no proper account for the energy of CDW deformations and their potential energy:

## A deeper problem

## By definition :

$$W_e = -\frac{\varepsilon(k, \omega) F^2}{8\pi} = -\frac{\varepsilon(k, \omega)}{8\pi e^2} \left( -\frac{\partial e\Phi}{\partial x} - \frac{\hbar v_F}{2} \frac{\partial^2 \varphi}{\partial x^2} \right)^2$$

For conducting electrons

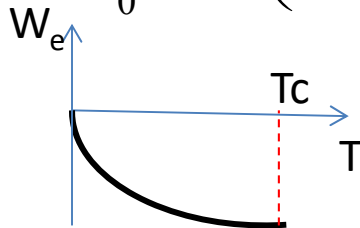
$$\varepsilon_{tot} = \varepsilon_{\Delta} + \varepsilon_n$$

$$\varepsilon_{\Delta} \propto \left( \frac{\omega_p}{\Delta} \right)^2 \Rightarrow W_e \propto \left( \frac{\partial^2 \phi}{\partial x^2} \right)^2 = k^4 \phi_k^2$$

$$\varepsilon_e = \frac{1}{\lambda^2 k^2} = \frac{\rho_n}{r_0^2 k^2} \quad \frac{1}{\lambda^2} = 4\pi e^2 \frac{dn}{d\mu} = 4\pi e^2 N_F \rho_n$$

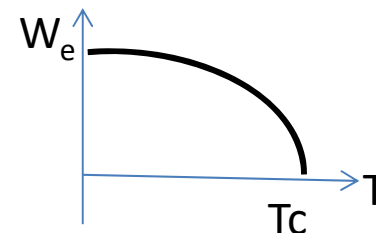
We obtain:

$$W_e^* = -\frac{\rho_n}{r_0^2} \frac{1}{8\pi} \left( e\Phi + \frac{\hbar v_F}{2} \frac{\partial \varphi}{\partial x} \right)^2$$



We expected to have:

$$W_e = \frac{\rho_c}{r_0^2} \frac{1}{8\pi} \left( e\Phi + \frac{\hbar v_F}{2} \frac{\partial \varphi}{\partial x} \right)^2$$



Resolution: the whole expression has been lost

*Chiral anomaly*

On top of perturbational part

$$\delta W = \frac{\hbar v_F}{4\pi} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{e\Phi}{\pi} \frac{\partial \phi}{\partial x}$$

$$W_e^* = -\frac{\rho_n}{r_0^2} \frac{1}{8\pi} \left( e\Phi + \frac{\hbar v_F}{2} \frac{\partial \phi}{\partial x} \right)^2$$

$$\zeta = \frac{\partial F}{\partial n}, \quad \rho_n = N_F^{-1} \frac{\partial n}{\partial \zeta}; \quad \rho_c = 1 - \rho_n$$

$$\varepsilon_e = \frac{1}{\lambda^2 k^2} = \frac{\rho_n}{r_0^2 k^2}$$

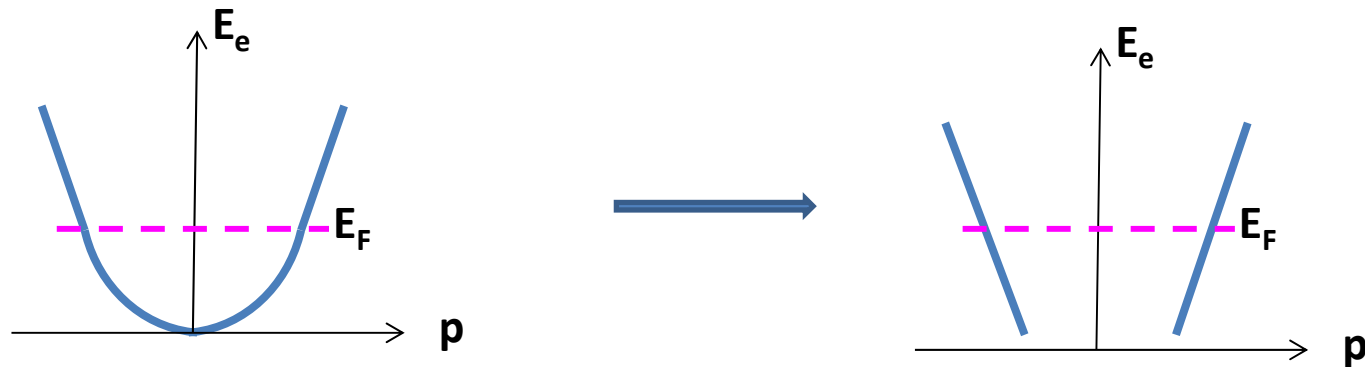
$\lambda(T)$  – screening length by normal carriers

$r_0$  – microscopic Tomas-Fermi scale

$$W_e = W_e^* + \delta W = \rho_c \frac{\hbar v_F}{4\pi} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{e}{\pi} \rho_c \Phi \frac{\partial \phi}{\partial x} - \frac{\rho_n (e\Phi)^2}{\pi \hbar v_F} - \frac{(\nabla \Phi)^2}{8\pi}$$

The contribution of normal carriers erases from the  $T=0$  anomalous action erasing it down to zero at  $T_c$  when  $A \sim \Delta \rightarrow 0$ ,  $\rho_c \sim A^2$

## Resolution of both problems: the anomaly in the effective action



With the linearized bottomless spectrum, no proper account for the energy of CDW deformations and their potential energy:

$$\delta W = \frac{\hbar v_F}{4\pi} \left( \frac{\partial \varphi}{\partial x} \right)^2 + \frac{e\Phi}{\pi} \frac{\partial \varphi}{\partial x} \quad \text{Chiral anomaly}$$

$$W_e = W_e^* + \delta W = \rho_c \frac{\hbar v_F}{4\pi} \left( \frac{\partial \varphi}{\partial x} \right)^2 + \frac{e}{\pi} \rho_c \Phi \frac{\partial \varphi}{\partial x} - \frac{\rho_n (e\Phi)^2}{\pi \hbar v_F} - \frac{(\nabla e\Phi)^2}{8\pi e^2}$$

The “superfluid density”  $\rho_c$  appears as  $\rho_c = 1 - \rho_n$  from partial compensation of the anomalous and perturbative contributions to the energy functional

In the metallic phase  $\rho_n = 1$  then  $\rho_c = 0$ ;

approaching from the CDW phase as  $\rho_c \sim \Delta^2 \sim A^2$

Resolution: the whole expression has been lost

*Chiral anomaly*

On top of perturbational

$$\delta W = \frac{\hbar v_F}{4\pi} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{e\Phi}{\pi} \frac{\partial \phi}{\partial x} \quad W_e^* = -\frac{\rho_n}{r_0^2} \frac{1}{8\pi} \left( e\Phi + \frac{\hbar v_F}{2} \frac{\partial \phi}{\partial x} \right)^2$$

$$W_e = W_e^* + \delta W = \rho_c \frac{\hbar v_F}{4\pi} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{e}{\pi} \rho_c \Phi \frac{\partial \phi}{\partial x} - \frac{\rho_n (e\Phi)^2}{\pi \hbar v_F} - \frac{(\nabla \Phi)^2}{8\pi}$$

$$\rho_n = 1 - \rho_c$$

The contribution of normal carriers erases from the T=0 anomalous action erasing it down to zero at  $T_c$  when  $A \sim \Delta \rightarrow 0$ ,  $\rho_c \sim A^2$

$$\zeta = \frac{\partial F}{\partial n}, \quad \rho_n = N_F^{-1} \frac{\partial n}{\partial \zeta}; \quad \rho_c = 1 - \rho_n$$

The chiral anomaly appears already above the CDW – in the normal metal

The most principle property of a conductor: expulsion of the electric field at the screening length  $r_0$

$$\frac{1}{r_0^2} = 4\pi e^2 \frac{dn(\zeta)}{d\zeta}$$

For a metal at  $T=0$  from statistical mechanics:

$$\frac{dn(\zeta)}{d\zeta} = \frac{dn}{d\varepsilon_F} = N_F \quad \text{- the DOS}$$

From quantum mechanics - the Tomas-Fermy procedure:

The nonlinearized Schroedinger eq. in WKB approximation

$$\psi(x) = \frac{C}{\sqrt[4]{E - e\Phi}} \exp\left(\pm i \int \sqrt{E - e\Phi(y)} dy\right)$$

$$n = \sum |\psi|^2 = \sum_E \frac{1}{\sqrt{E - e\Phi}} \sim \sqrt{\varepsilon_F - e\Phi} \quad \delta n \sim \delta \Phi$$

When we linearize the spectrum and decompose the wave function

$$\frac{p^2}{2m} - \frac{p_F^2}{2m} \cong \pm \hbar v_F k \quad \psi = \psi_+ e^{ik_F x} + \psi_- e^{-ik_F x}$$

$$\mp i \partial_x \psi_{\pm} + e \Phi \psi_{\pm} = E \psi_{\pm} \quad \psi_{\pm} = C \exp\left(\pm i \int dx (E - e \Phi(x))\right)$$

With  $C = \text{const}$  – the potential affects only the phase  
but no more the amplitude of the wave function

$$n = \sum |\psi_{\pm}|^2 = \text{const}$$

By the necessary but premature linearization we lose  
the possibility to change the density  $|\psi|^2$  for any wave function

No density response to the potential already  
at one-particle level, hence no field screening

The chiral anomaly is invoked to restore the correct response

## Local energy functional

$$\psi = A \exp(i\varphi); \quad A = \Delta/\Delta_0$$

$$W \{ \phi, \Phi, n_{in}, A \} = \frac{\hbar v_F}{4\pi} \left[ \phi_x^2 + \alpha A^2 \phi_y^2 \right] + C \frac{\hbar v_F}{4\pi} \left[ \left| \frac{\partial A}{\partial x} \right|^2 + \beta^2 \left| \frac{\partial A}{\partial y} \right|^2 \right] +$$

Expect  $A^2$  –  
actually 1.

Non analytic in  $\Psi$

Both terms come from  
the chiral anomaly.

$$\frac{\phi_x}{\pi} \Phi + \left( \Phi + \frac{\hbar v_F}{2} \phi_x \right) n_{in} - \frac{\varepsilon_h}{8\pi} (\nabla \Phi)^2 + F(A, n_{in})$$

$$\phi_i = \partial \phi / \partial x_i$$

Expressions for total density and current conserve number of particles

$$n = \frac{1}{\pi} \partial_x \varphi + n_{in}, \quad j = -\frac{1}{\pi} \partial_t \varphi + j_{in} \Rightarrow \frac{dn}{dt} = 0$$

+ need a mechanism for  $n_{in}$ ,  $j_{in}$  to compensate  $\partial \varphi$  at  $A \rightarrow 0$

$$n_c = \frac{A^2}{\pi} \frac{\partial \varphi}{\partial x} \quad j_c = -\frac{A^2}{\pi} \frac{\partial \varphi}{\partial t} \quad \text{for } A = \text{cnst}$$

True equations are not analytic in  $\Psi$ :  
phase gradients are not multiplied by  $A^2$

$$\frac{\partial}{\partial x} \left( \frac{\partial \varphi}{\partial x} + 2\Phi + \pi(n_e - n_h) \right) + \alpha \frac{\partial}{\partial y} \left( A^2 \frac{\partial \varphi}{\partial y} \right) = \gamma_\varphi A^2 \frac{\partial \varphi}{\partial t}$$

$$\nabla^2 A + \alpha A \left( \frac{\partial \varphi}{\partial y} \right)^2 + \frac{\partial F}{\partial A} = -\gamma_A \frac{\partial A}{\partial t}$$

$$-r_0^2 \nabla^2 \Phi = \frac{\partial \varphi}{\partial x} + \pi(n_e - n_h + n_{ex})$$

$$\nabla \hat{\sigma} \nabla \mu = \partial_t n = \frac{1}{4\pi} \frac{1}{r_0^2} \rho_n \partial_t \zeta$$

$$\mu = \zeta + \Phi + \partial_x \varphi / 2$$

Former G-L like equations

$$\nabla A^2 \nabla \varphi + \frac{\partial}{\partial x} A^2 \Phi = \gamma_\varphi A^2 \frac{\partial \varphi}{\partial t}$$

$$\nabla^2 A + A(\nabla \varphi)^2 + \frac{\partial F}{\partial A} = -\gamma_A \frac{\partial A}{\partial t}$$

$$-r_0^2 \nabla^2 \Phi = A^2 \frac{\partial \varphi}{\partial x} + n_{ex}$$

$$-\nabla [\sigma \nabla (\zeta + \Phi)] + \frac{\partial n}{\partial t} = 0$$

*All energies are in units of  $\Delta$*

The terms with  $\partial_x \varphi$  - the elastic and the Coulomb energy of condensed electrons of the CDW, look really counterintuitive:

their shown factors  $l$  are anticipated (in the conventional GL scheme) to be  $A^2$  – at least we expect them vanish together with  $A$ .

The wonder of this approach is that this would happen indeed after elimination of  $n_{in}$  from equations of motion generated by this functional.

That is proved but cannot be done explicitly rather than with a help of additional approximations.

Enough for us but bad for the computer which cannot see explicitly the compensation of divergent phase gradients while approaching the vortex core

## Possible simplifications and explicitness

Infinite conductivity: - a bridge to the naive GL eqs.  $\mu = \zeta + \Phi + \partial_x \varphi / 2 \equiv 0$

$$\rho_c \partial_x^2 \varphi = r_0^2 \nabla^2 E_x - \rho_n E_x \quad \text{Poisson eq.}$$

LHS resembles the static effective charge  
 $n_c = A^2 \partial_x \varphi / \pi$  - identifying  $\rho_c$  and  $A^2$

But instead:  $\partial_x n_c = \rho_c \partial_x^2 \varphi / \pi$

screening of  $E_x$  with a  
 standard local screening  
 length  $l^2 = r_0^2 / \rho_n$

$$-\rho_c E_x + \left( \rho_c \partial_x^2 + \partial_y (\alpha A^2 \partial_y) - \gamma_\varphi \partial_t \right) \varphi = 0 \quad \text{Phase eq.}$$

Resembles GL with  $\rho_c$  as  $A^2$  but  
 with no differentiation of the amplitude :

Not like variational eqs.

$\rho_c \partial_x \Phi$  instead of  $\partial_x (A^2 \Phi)$

$\rho_c \partial_x^2 \varphi$  instead of  $\partial_x (A^2 \partial_x \varphi)$

The limit of the local electro-neutrality  $r_o \rightarrow 0$   
together with the infinite normal conductivity.

$$\partial_x \Phi + (\pi N_F)^{-1} \nabla_{\perp} A^2 \nabla_{\perp} \varphi - \gamma_{\varphi} \partial_t \varphi = 0, \quad \Phi = (n/N_F - \zeta)$$

$$\partial_x \varphi + \pi n = 0$$

Curiously, no commonly assumed longitudinal phase rigidity  $\propto \partial_x^2 \varphi$

It is hidden in the term  $\partial_x \Phi$  implicitly, via relations.

$$\frac{\rho_c}{\rho_n} \partial_x^2 \varphi + \kappa_{\perp} \nabla_{\perp} (A^2 \nabla_{\perp} \varphi) - \gamma_{\varphi} \partial_t \varphi = \pi N_F \frac{\partial \zeta}{\partial A} \partial_x A$$

Coulomb hardening (rem. Kirova talk) looks intuitive,  
but **where is the driving force?**

The drive comes only from the boundary conditions for  $\Phi$  transferred to the phase via the local relations of  $\Phi$  and  $\partial \varphi$  mediated by  $n$ .

## Electro-neutrality at a finite normal conductivity at D=1:

Static elastic force:  
gradient of the normal chem. potential

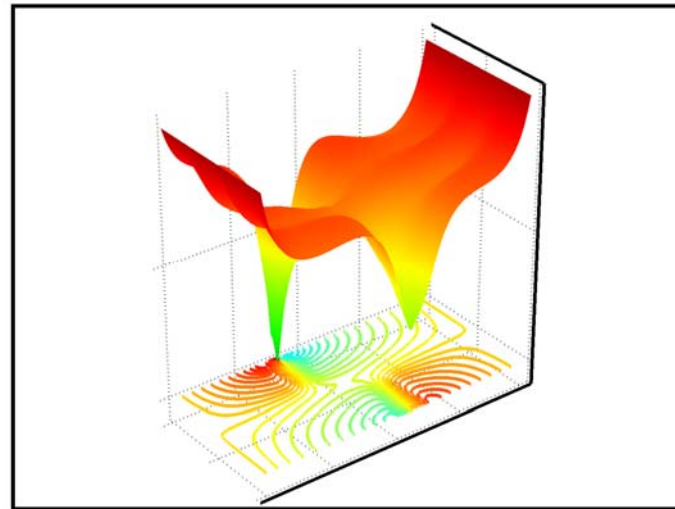
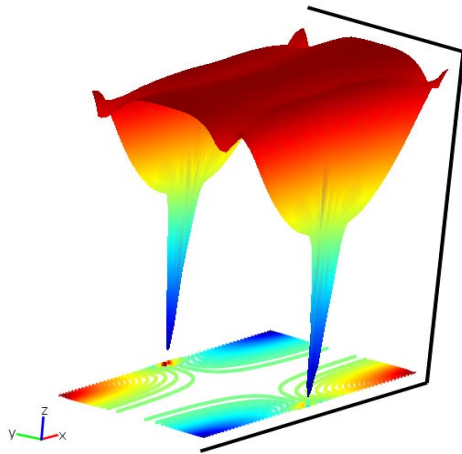
$$\frac{1}{\pi} \left( \frac{1}{\sigma_{CDW}} + \frac{1}{\sigma_n} \right) \partial_t \varphi + \partial_x \left( \frac{1}{\pi} \partial_x \varphi + \zeta \right) = \frac{-1}{\sigma_n} J(t)$$

Serial CDW and normal resistivities

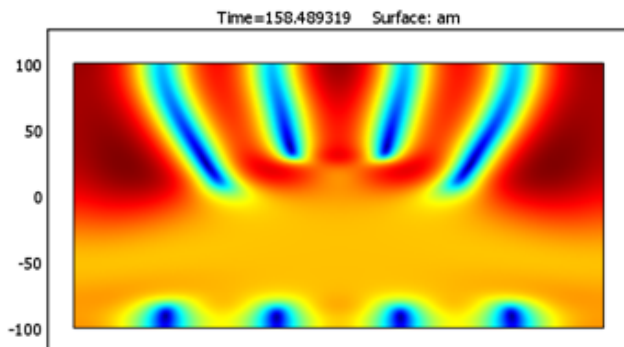
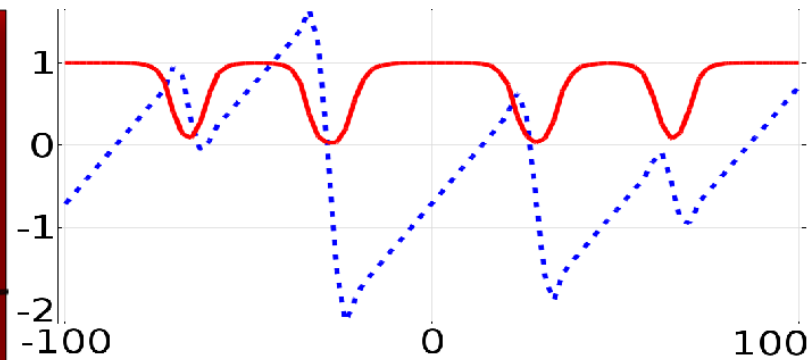
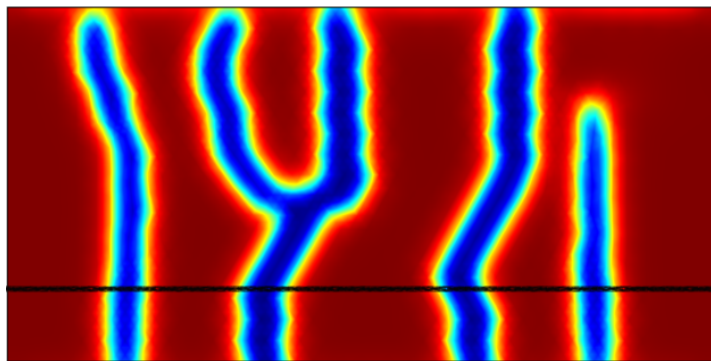
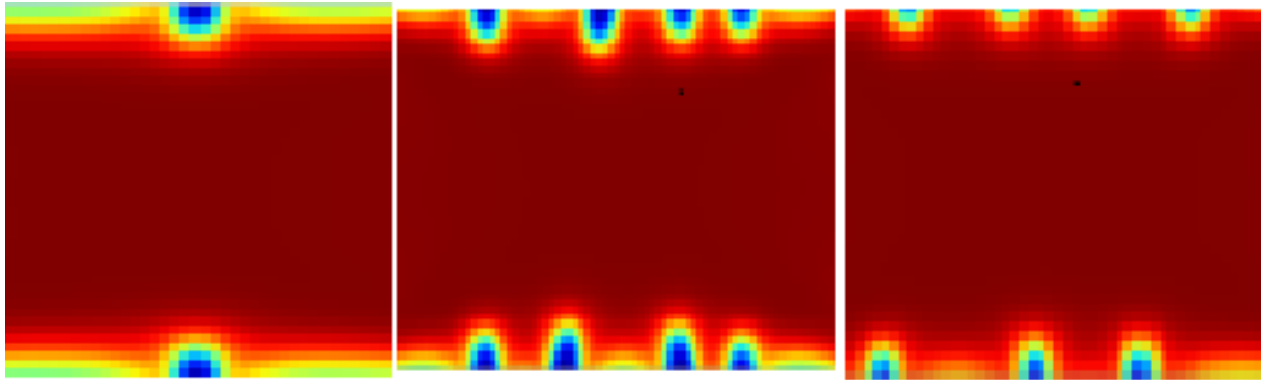
Current driving force:  
(Total current)/(normal conduct.)

$$\frac{\rho_c}{\rho_n} (\partial_x \varphi)^2$$

Nonanalytic dependence on the amplitude requires new more complicated numerical studies.



We still can run up to nucleation of vortices at a surface.  
But we cannot trace proliferation of vortices as before.  
A price for no explicit compensation of diverging  $\partial\phi$  by vanishing  $\mathbf{A}^2$



Former results for the GL model:  
Also flashes of  $\mathbf{A}=0$  but their lines  
can relax leaving the leading vortex alone.

## Conclusion and perspective.

- Chiral transformations with account for chiral anomaly were applied to the sliding CDW model
- Two-fluid hydrodynamics was constructed for the order parameter and the normal liquid
- Topologically nontrivial dynamics appears under applied fields or charge injection
- Spontaneous space-time vorticity under the driving current was modeled numerically some under simplifications, periodic sequence of amplitude nodes with  $2\pi$  phase increments was demonstrated
- The numeric procedure needs to be stabilized for the nonanalytic eqs.
- The problems of glide and climb should be considered

Beyond quasi-1D and CDWs, e.g. for polaronic crystals

$\mathbf{u}$  – vector of unit cell displacements

$\nu$  – units' filling factor

$$n_c = -\nu \nabla \cdot \mathbf{u}; \quad \mathbf{j}_c = \nu \partial_t \mathbf{u}$$

$$\frac{dn_c}{dt} = \partial_t n_c + (\nabla \cdot \mathbf{j}_c) = (\nabla \nu \cdot \partial_t \mathbf{u}) - \partial_t \nu (\nabla \cdot \mathbf{u}) \neq 0$$

Need to bridge the elastic theory and inter-cell kinetics

The most common technical tool in fermionic systems is the linearization of the spectrum near the Fermi surface

$$\varepsilon(\vec{p} + \vec{k}) - \varepsilon(\vec{p}) \rightarrow \vec{v}_F(\vec{p})\vec{k} \rightarrow v_F \vec{n}(\vec{p})\vec{k} \rightarrow v_F k \quad \text{if } D=1$$

In D=1 that becomes even more useful, even conceptual, but potentially harmful step: division of the fermionic Hilbert space into its right- and left- moving components.



“... We show that Luttinger did not solve his model properly because of the paradoxical fact that the density operator commutators, which always vanish for any finite number of particles, no longer vanish in the field-theoretic limit of a filled Dirac sea.”

*D. Mattis and E. Lieb, J. Math. Phys. 6, 304 (1965)*

*In response to the “would become the Luttinger Liquid”*

*J. M. Luttinger, J. Math. Phys. 4, 1154 (1963)*

For us, in condensed matter, the answer is just the basic foundation of the quantum hydrodynamics, model independently:

$$\rho_+ + \rho_- = \rho, \quad \rho_+ - \rho_- = j \quad (v_F = 1)$$

$$[\rho, j] = -i \nabla \delta(r - r') \neq 0$$

$$\Psi^\dagger \hat{\epsilon} \Psi \rightarrow \rho_+^2 + \rho_-^2 \quad \rho_\pm = \Psi_\pm^\dagger \Psi_\pm$$

For a universal long-wave Hamiltonian of fermions in the potential  $\Phi$

$$H_{LR} = K/2 \rho^2 + \rho \Phi$$

The commutator  $[\rho, j] = -i \nabla \delta(r - r')$

leads to the “chiral anomaly” of the “axial current” of the 1+1 quantum field theory:

$$\partial_t j + K \partial_x \rho = -\partial_x \Phi$$

The “anomalous” – missed in the perturbation theory – RHS actually restores the most basic property of the ideal metal:  $\mathbf{j} = \mathbf{E}t$  and of any conductor:  $\rho = -\Phi/K = -\Phi N_F$

Why has it been missed ?