

Asymmetric higher-harmonic SQUID as a Josephson diode

[Phys. Rev. B **106**, 134514 (2022)]

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Ա.Ալիխանյանի Անվան Ազգային Գիտական Լաբորատորիա
A.Alikhanyan National Scientific Laboratory



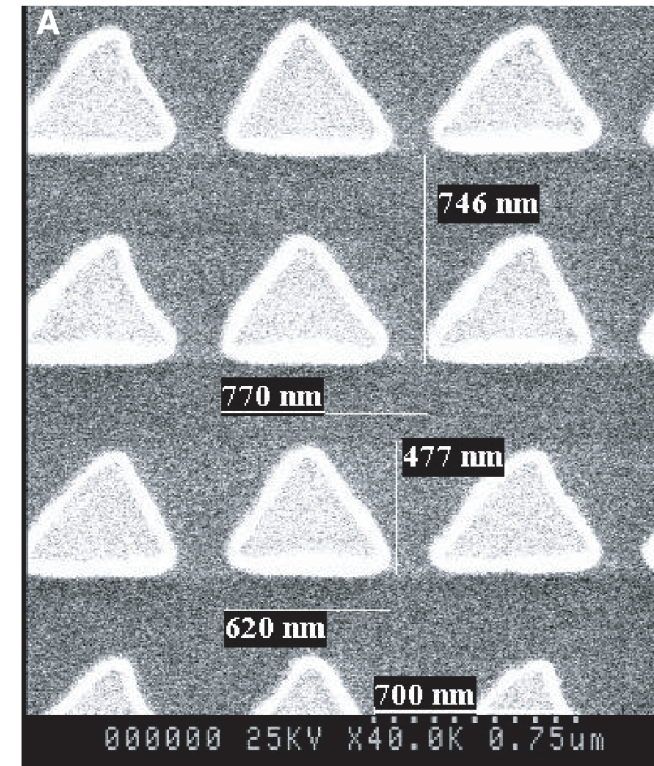
Superconducting ratchet effect

Science **302**, 1188 (2003)

A Superconducting Reversible Rectifier That Controls the Motion of Magnetic Flux Quanta

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J. V. Anguita,⁴ R. García,⁴ J. L. Vicent^{1*}

We fabricated a device that controls the motion of flux quanta in a niobium superconducting film grown on an array of nanoscale triangular pinning potentials. The controllable rectification of the vortex motion is due to the asymmetry of the fabricated magnetic pinning centers. The reversal in the direction of the vortex flow is explained by the interaction between the vortices trapped on the magnetic nanostructures and the interstitial vortices. The applied magnetic field and input current strength can tune both the polarity and magnitude of the rectified vortex flow. Our ratchet system is explained and modeled theoretically, taking the interactions between particles into consideration.



Superconducting diode effect

PHYSICAL REVIEW B **72**, 064509 (2005)

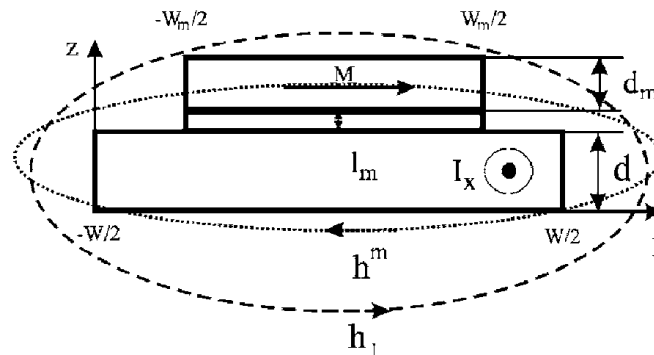
Considerable enhancement of the critical current in a superconducting film by a magnetized magnetic strip

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(Received 8 April 2005; revised manuscript received 21 June 2005; published 9 August 2005)

We show that a magnetic strip on top of a superconducting strip magnetized in a specified direction may considerably enhance the critical current in the sample. At fixed magnetization of the magnet we observed the diode effect—the value of the critical current depends on the direction of the transport current. We explain these effects by a influence of the nonuniform magnetic field induced by the magnet on the current distribution in the superconducting strip. The experiment on a hybrid Nb/Co structure confirmed the predicted variation of the critical current with a changing value of magnetization and direction of the transport current.



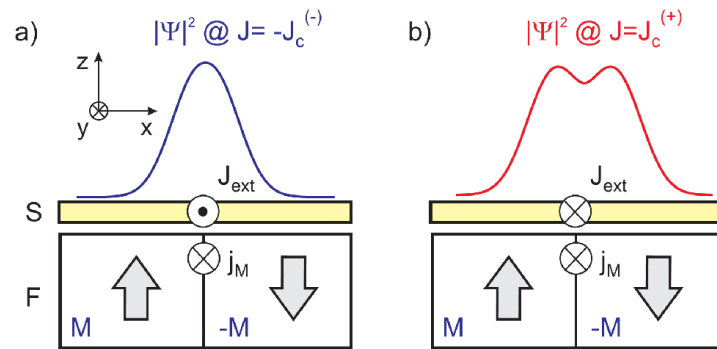
Superconducting diode effect

J. Phys.: Condens. Matter **26** (2014) 095702 (7pp)

doi:10.1088/0953-8984/26/9/095702

The diode effect induced by domain-wall superconductivity

M A Silaev¹, A Yu Aladyshkin^{1,2}, M V Silaeva³ and A S Aladyshkina³



Parity-breaking vector:

$$\mathbf{j}_M = c \nabla \times \mathbf{M}$$

Figure 1. Schematic view of the hybrid system, consisting of a superconducting film (S) and ferromagnetic (F) substrate with a single domain wall. The grey arrows show the local magnetization and the orientation of the magnetic self-field above the magnetic domains. The parity-breaking vector $\mathbf{j}_M$ given by equation (1) is oriented along the y-axis; the direction of the external current is antiparallel to $\mathbf{j}_M$ (panel a) and parallel to $\mathbf{j}_M$ (panel b). The OP profiles $|\Psi|^2$ for the DWS channel, carrying maximal supercurrents in each direction, are shown schematically.

Superconducting diode effect

PNAS **119**, e2119548119 (2022):

Supercurrent diode effect and finite-momentum superconductors

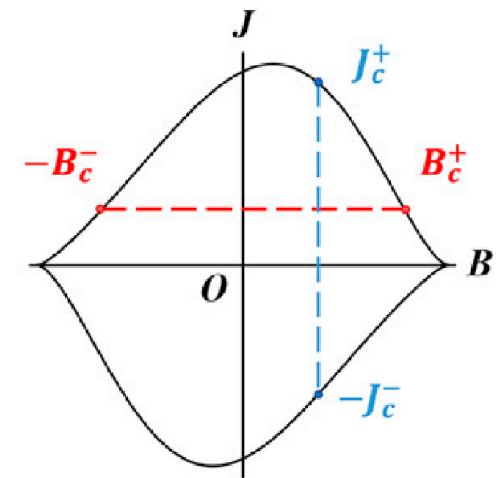
Noah F. Q. Yuan^{a,1}  and Liang Fu^{b,1}

When both inversion and time-reversal symmetries are broken, the critical current of a superconductor can be nonreciprocal. In this work, we show that, in certain classes of two-dimensional superconductors with antisymmetric spin-orbit coupling, Cooper pairs acquire a finite momentum upon the application of an in-plane magnetic field, and, as a result, critical currents in the direction parallel and antiparallel to the Cooper pair momentum become unequal. This supercurrent diode effect is also manifested in the polarity dependence of in-plane critical fields induced by a supercurrent. These nonreciprocal effects may be found in polar SrTiO₃ film, few-layer MoTe₂ in the T_d phase, and twisted bilayer graphene in which the valley degree of freedom plays a role analogous to spin.

$$\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{Q}\mathbf{r}}$$

Ingredients:

- broken inversion symmetry (P-symmetry) – spin-orbit coupling
- broken time-reversal symmetry (T-symmetry) – magnetic (Zeeman) field



Superconducting diode effect (experiment)

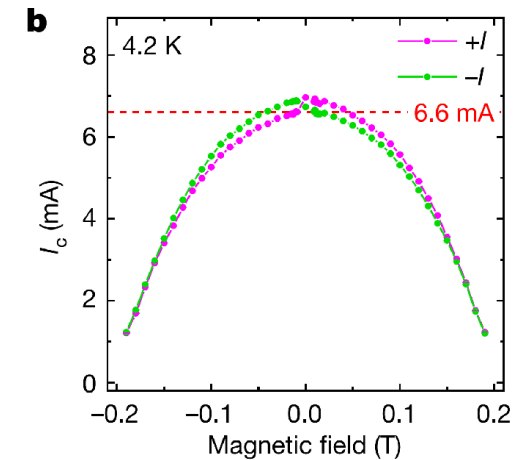
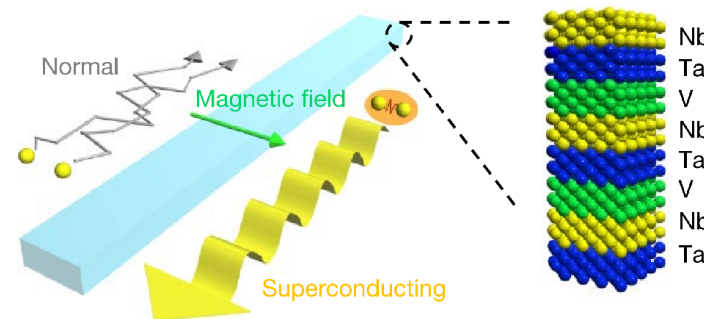
Observation of superconducting diode effect

<https://doi.org/10.1038/s41586-020-2590-4>

Received: 14 March 2020

[Nature (2020)]

Fuyuki Ando¹, Yuta Miyasaka¹, Tian Li¹, Jun Ishizuka², Tomonori Arakawa^{3,4}, Yoichi Shiota¹, Takahiro Moriyama¹, Youichi Yanase² & Teruo Ono^{1,4}✉

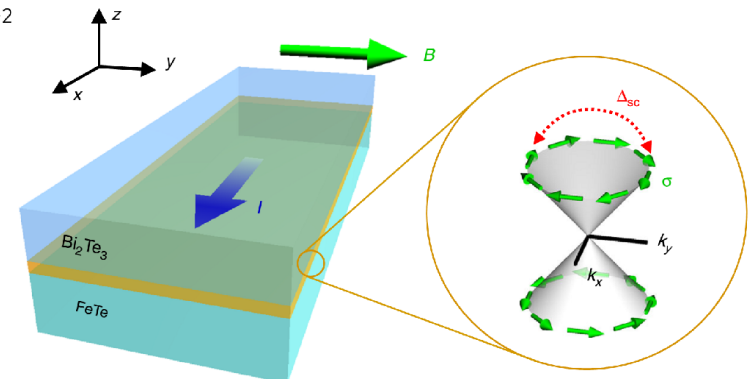


Nonreciprocal charge transport at topological insulator/superconductor interface

Kenji Yasuda^{1,5}, Hironori Yasuda¹, Tian Liang², Ryutaro Yoshimi², Atsushi Tsukazaki^{1b}, Kei S. Takahashi², Naoto Nagaosa^{1,2}, Masashi Kawasaki^{1,2} & Yoshinori Tokura^{1b}

[Nature Communications (2019)]

$$V = R_0 I (1 + \gamma (\mathbf{B} \times \mathbf{z}) \cdot \mathbf{I})$$



Josephson diode effect (JDE)

PHYSICAL REVIEW B **89**, 195407 (2014)

Anomalous Josephson effect induced by spin-orbit interaction and Zeeman effect in semiconductor nanowires

Tomohiro Yokoyama Mikio Eto Yuli V. Nazarov

PHYSICAL REVIEW B **103**, 144520 (2021)

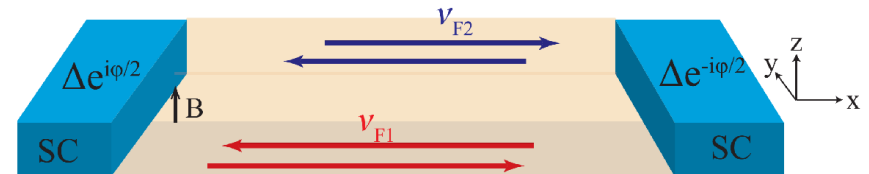
Geometry controlled superconducting diode and anomalous Josephson effect triggered by the topological phase transition in curved proximitized nanowires

A. A. Kopasov ^{1,2,*} A. G. Kutlin ³ and A. S. Mel'nikov^{1,2,4}

Asymmetric Josephson effect in inversion symmetry breaking topological materials

Chui-Zhen Chen,¹ James Jun He,¹ Mazhar N. Ali,² Gil-Ho Lee,^{3,*} Kin Chung Fong,^{4,†} and K. T. Law^{1,‡}

PHYSICAL REVIEW B **98**, 075430 (2018)



Ingredients:

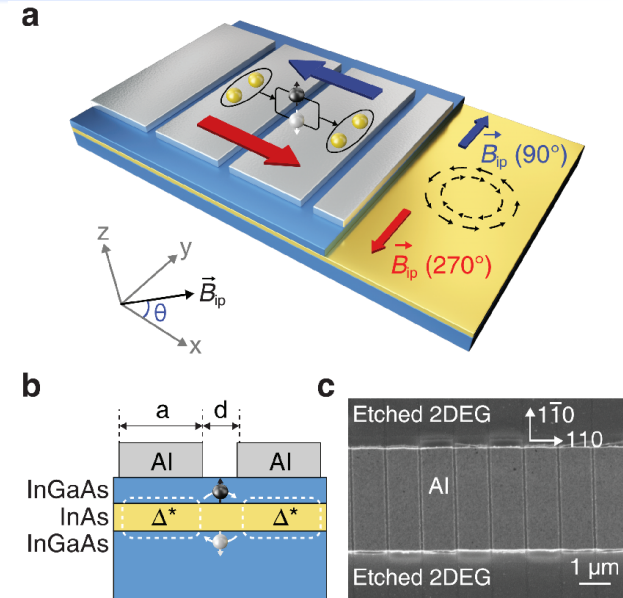
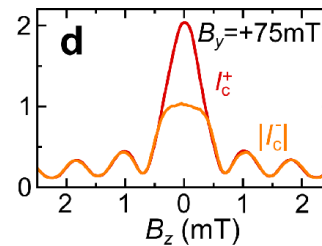
- broken inversion symmetry (P-symmetry)
- broken time-reversal symmetry (T-symmetry) – magnetic field

Josephson diode effect (experiment)

Supercurrent rectification and magnetochiral effects in symmetric Josephson junctions

Christian Baumgartner^{1,8}, Lorenz Fuchs^{1,8}, Andreas Costa², Simon Reinhardt¹, Sergei Gronin^{3,4}, Geoffrey C. Gardner^{3,4}, Tyler Lindemann^{4,5}, Michael J. Manfra^{3,4,5,6,7}, Paulo E. Faria Junior², Denis Kochan², Jaroslav Fabian², Nicola Paradiso¹ and Christoph Strunk¹

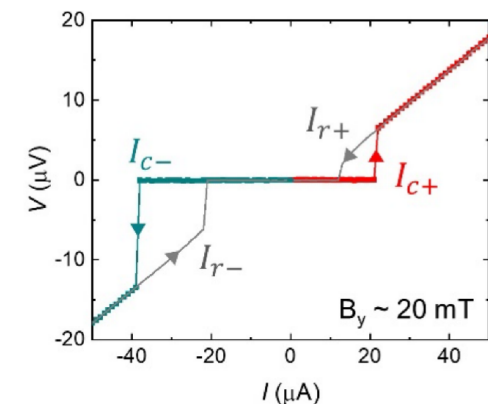
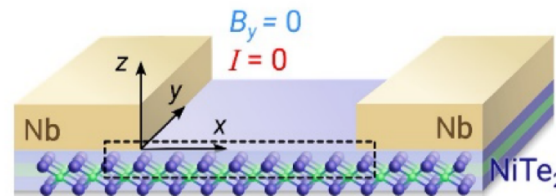
Nature Nanotechnology (2022)



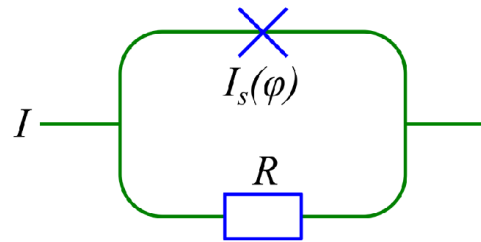
Josephson diode effect from Cooper pair momentum in a topological semimetal

Banabir Pal^{1,4}, Anirban Chakraborty^{1,4}, Pranava K. Sivakumar^{1,4}, Margarita Davydova^{2,4}, Ajesh K. Gopi¹, Avanindra K. Pandeya¹, Jonas A. Krieger¹, Yang Zhang², Mihir Date¹, Sailong Ju³, Noah Yuan², Niels B. M. Schröter¹, Liang Fu² and Stuart S. P. Parkin¹

Nature Physics (2022)



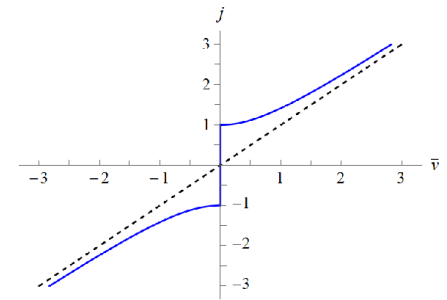
Resistively-shunted junction (RSJ) model



$$\frac{V}{R} + I_s(\varphi) = I, \quad V = \frac{\hbar}{2e} \frac{\partial \varphi}{\partial t}$$

$I_s(\varphi) = I_1 \sin \varphi$ — sinusoidal current-phase relation (CPR)

Units: $\omega_0 = 2eI_1R/\hbar$, $\tau = \omega_0 t$, $j = I/I_1$, $v = V/I_1R$



Resistively-shunted junction (RSJ) model: $v = \dot{\varphi}$
 $\dot{\varphi} + \sin \varphi = j$

[Stewart (1968), McCumber (1968)]

Solution at $j > 1$:

[Aslamazov, Larkin (1969)]

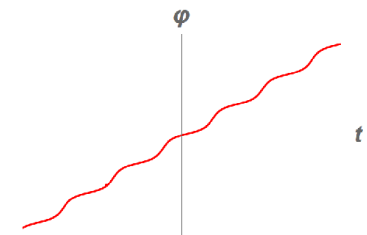
$$\varphi(\tau) = 2 \arctan \left[\frac{1 + \nu \tan(\nu\tau/2)}{\sqrt{\nu^2 + 1}} \right] + 2\pi n, \quad n(\tau) = \lfloor \nu\tau/2\pi \rfloor$$

$$v(\tau) = \frac{j^2 - 1}{j + \cos(\nu\tau - \alpha)}$$

where $\nu = \sqrt[3]{j^2 - 1}$, $\cos \alpha = 1/j$, $\sin \alpha = \sqrt[3]{j^2 - 1}/j$

$$\sqrt[3]{j^2 - 1} \equiv \text{sgn}(j) \sqrt{j^2 - 1}$$

General relation: $\bar{v} = \nu$



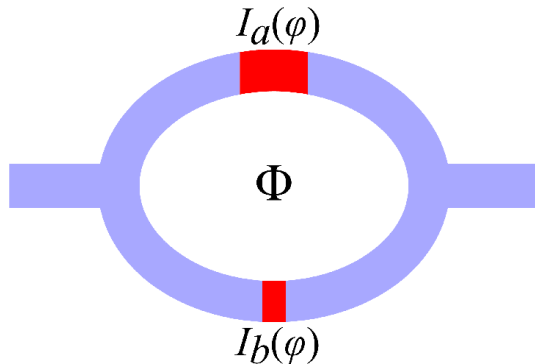
Higher Josephson harmonics

$$I_s(\varphi) = \sum_{n=1}^{\infty} I_n \sin n\varphi \quad \text{— nonsinusoidal CPR with higher harmonics}$$

RSJ equation: $\dot{\varphi} + J(\varphi) = j, \quad J(\varphi) = I_s(\varphi)/I_1$

General relation: $\bar{\nu} = \nu$

SQUID



$$\varphi_a - \varphi_b = \phi, \quad \phi = 2\pi \frac{\Phi}{\Phi_0}$$

CRP of the SQUID: $I_s(\varphi, \phi) = I_a(\varphi_a) + I_b(\varphi_b) = I_a(\varphi + \phi/2) + I_b(\varphi - \phi/2)$

Sinusoidal case: $I_s(\varphi, \phi) = I_{a1} \sin(\varphi + \phi/2) + I_{b1} \sin(\varphi - \phi/2) = I_1(\phi) \sin(\varphi + \gamma)$

where $I_1(\phi) = \sqrt{I_{a1}^2 + I_{b1}^2 + 2I_{a1}I_{b1} \cos \phi}$

$$\tan \gamma = \frac{I_{a1} - I_{b1}}{I_{a1} + I_{b1}} \tan \frac{\phi}{2}$$

Critical currents in two directions: $I_c^+ = I_c^-$

More generally, this is so

(a) in symmetric SQUIDS with arbitrary $I_a(\varphi) = I_b(\varphi)$

(b) when $I_a(\varphi)$ and $I_b(\varphi)$ contain the same single harmonic (with arb. amplitudes)

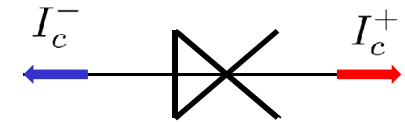
Asymmetric higher-harmonic SQUID as a Josephson diode

General case: $I_c^+ \neq I_c^-$

“Minimal model”:

$$I_a(\varphi) = I_{a1} \sin \varphi$$

$$I_b(\varphi) = I_{b1} \sin \varphi + I_{b2} \sin 2\varphi$$



Effective Josephson junction with

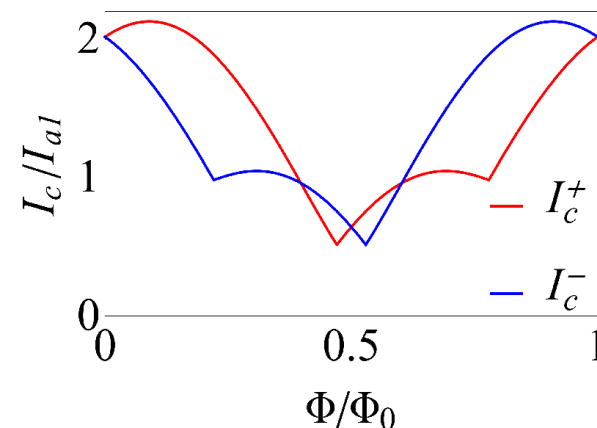
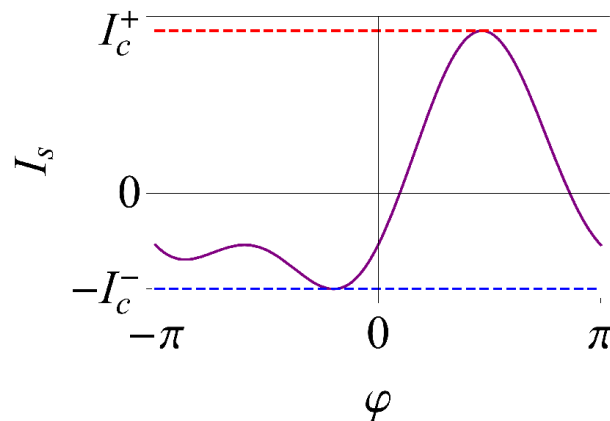
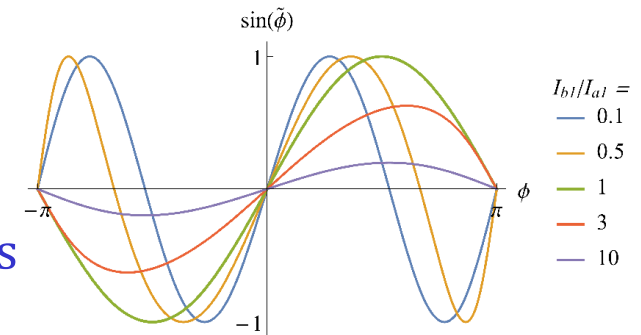
$$J(\varphi) = \sin \varphi + A \sin(2\varphi - \tilde{\phi})$$

where

$$A(\phi) = I_{b2}/I_1(\phi), \quad \tilde{\phi} = \phi + 2\gamma(\phi)$$

N.B.:

- broken TRS due to magnetic flux in the SQUID
- *any* asymmetry between the two Josephson junctions are not sufficient for the diode effect



CVC: harmonic perturbation theory in the limit of $|j| \gg 1$

$$\varphi(\tau) = \nu\tau + \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\nu\tau + b_n \sin n\nu\tau)$$

$$\nu = \sum_{n=0}^{\infty} \nu^{(n)}, \quad \nu^{(n)} \sim 1/j^{n-1}$$

$$a_n = \sum_{k=0}^{\infty} a_n^{(k)}, \quad a_n^{(k)} \sim 1/j^{n+k}$$

$$b_n = \sum_{k=0}^{\infty} b_n^{(k)}, \quad b_n^{(k)} \sim 1/j^{n+k}$$

Current-voltage characteristic (CVC) in the sinusoidal case:

$$\bar{v} = \nu = j - \frac{1}{2j} - \frac{1}{8j^3}$$

“Minimal model”: $\dot{\varphi} + \sin \varphi + A \sin(2\varphi - \tilde{\phi}) = j$

CVC (at arbitrary A):

$$\bar{v} = \nu = j - \frac{1 + A^2}{2j} - \frac{3A \sin(\tilde{\phi})}{4j^2} \quad \text{— asymmetry (JDE)}$$

CVC: perturbation theory in the limit of $|A| \ll 1$

Perturbation theory with feedback (à la [Thompson (1973)]):

$$\varphi(\tau) = f_0(\tau) + Af_1(\tau) + A^2f_2(\tau) + \dots$$

$$j = i_0 + Ai_1 + A^2i_2 + \dots \quad \text{— this feedback should ensure that } f_n(\tau) \text{ with } n = 1, 2, \dots \text{ are limited functions}$$

Order by order: $\dot{f}_0 + \sin f_0 = i_0$

$$\dot{f}_n + f_n \cos f_0 = D_n, \quad n = 1, 2, \dots$$

$$\bar{v} = \overline{\dot{f}_0} = \sqrt[3]{i_0^2 - 1}$$

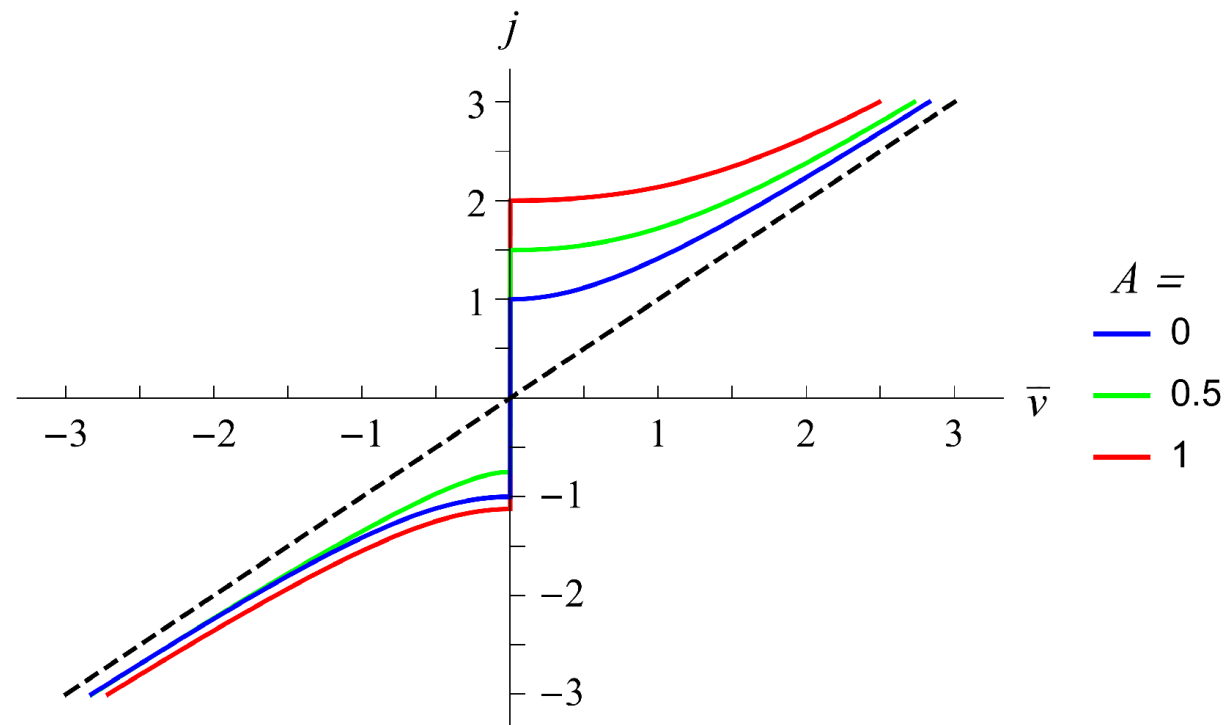
CVC $j(\bar{v})$ becomes $(i_0 + Ai_1 + A^2i_2 + \dots)$ vs. $\sqrt[3]{i_0^2 - 1}$

$$\overline{D_n(\tau) \exp \int_0^\tau \cos f_0(t) dt} = 0 \longrightarrow i_n$$

CVC (at arbitrary j):

$$\bar{v} = \sqrt[3]{j^2 - 1} - A \frac{2 \sqrt[3]{j^2 - 1}^3 - 2j^3 + 3j}{\sqrt[3]{j^2 - 1}} \sin \tilde{\phi} \quad \text{— asymmetry (JDE)}$$

CVC: numerical results



Diode effect:

$$j_c^+ \neq j_c^-$$
$$j(-\bar{v}) \neq -j(\bar{v})$$

Shapiro spikes

Voltage-driven junction: $v(\tau) = v_{\text{dc}} + v_{\text{ac}} \cos \Omega\tau$

$$\varphi(\tau) = \varphi_0 + v_{\text{dc}}\tau + (v_{\text{ac}}/\Omega) \sin \Omega\tau$$

General CPR: $J(\varphi) = \sum_{k=1}^{\infty} A_k \sin(k\varphi + \Phi_k)$

$$J(\tau) = \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} (-1)^n A_k J_n \left(\frac{kv_{\text{ac}}}{\Omega} \right) \sin [k\varphi_0 + \Phi_k + (kv_{\text{dc}} - n\Omega)\tau]$$

$$\bar{j} = v_{\text{dc}} + \bar{J} = v_{\text{dc}} + \begin{cases} \sum'_{k,n} (-1)^n A_k J_n \left(\frac{kv_{\text{ac}}}{\Omega} \right) \sin(k\varphi_0 + \Phi_k), & \text{if } v_{\text{dc}} = \frac{n}{k}\Omega, \\ 0, & \text{else.} \end{cases}$$

Shapiro spikes in the minimal model

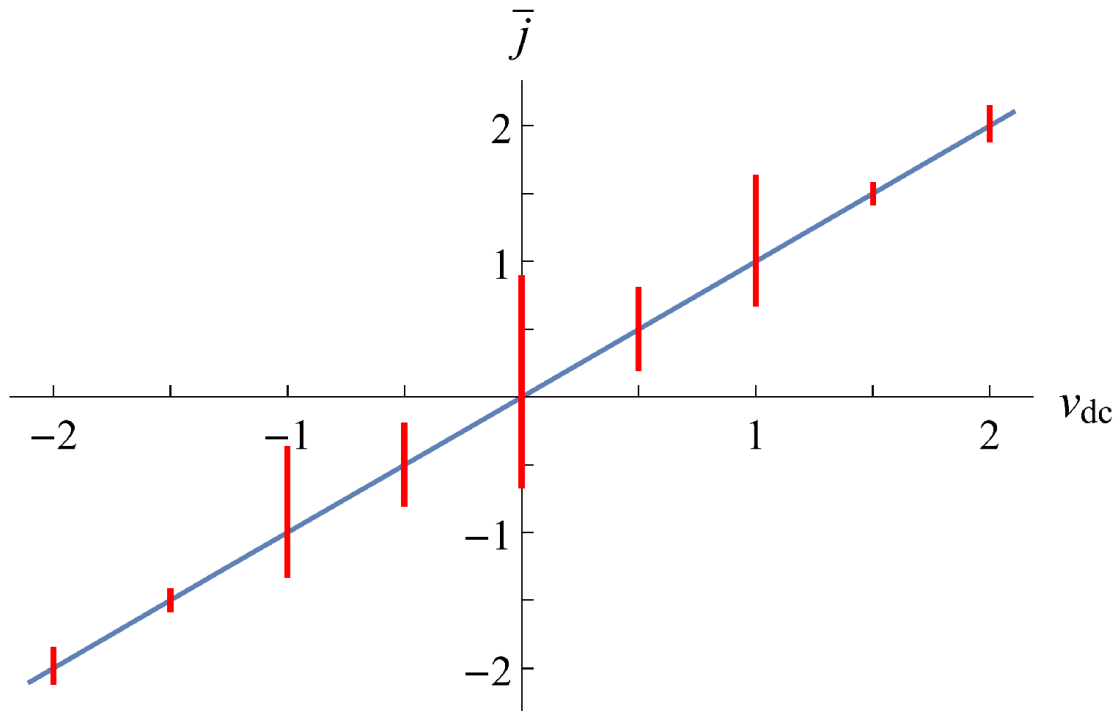
$v_{\text{dc}} = (n/2)\Omega$ — single contribution, symmetric spikes:

$$j_{\text{max}(\text{min})} = v_{\text{dc}} \pm |AJ_n(2v_{\text{ac}}/\Omega)|$$

$v_{\text{dc}} = n\Omega$ — interplay of two contributions: $(k, n) = (1, n)$ or $(2, 2n)$:

$$j_{\text{max}(\text{min})} = v_{\text{dc}} + \max_{\varphi_0}(\min) \left[J_{|n|}(v_{\text{ac}}/\Omega) \sin \varphi_0 + \textcolor{red}{A} J_{|2n|}(2v_{\text{ac}}/\Omega) \sin(2\varphi_0 - \textcolor{red}{\tilde{\phi}}) \right]$$

— asymmetric spikes, JDE



Diode effect:

$$j_c^+ \neq j_c^-$$

$$\bar{j}(-v_{\text{dc}}) \neq -\bar{j}(v_{\text{dc}})$$

Shapiro steps

Current-driven junction: $\dot{\varphi} + J(\varphi) = j_{\text{dc}} + j_{\text{ac}} \cos(\Omega\tau + \beta)$

Perturbation theory with feedback (à la [Thompson (1973)]), small j_{ac} :

$$\varphi(\tau) = \varphi_0(\tau) + \varphi_1(\tau) + \varphi_2(\tau) + \dots$$

$$j_{\text{dc}} = j_0 + j_1 + j_2 + \dots \quad \text{— this feedback should ensure that } \varphi_n(\tau) \text{ with } n = 1, 2, \dots \text{ are limited functions}$$

Order by order: $\dot{\varphi}_0 + j_s(\varphi_0) = j_0$

$$\dot{\varphi}_n + [\partial J(\varphi_0)/\partial \varphi_0] \varphi_n = D_n, \quad n = 1, 2, \dots$$

CVC $j_{\text{dc}}(\bar{v})$ becomes $(j_0 + j_1 + j_2 + \dots)$ vs. $\overline{\dot{\varphi}_0}$

$$\overline{D_n(\tau)/\dot{\varphi}_0(\tau)} = 0 \longrightarrow j_n$$

Shapiro steps, first order with respect to j_{ac}

Sinusoidal junction: $\overline{[j_1 + j_{ac} \cos(\Omega\tau + \beta)] [i_0 + \cos(\nu_0\tau - \alpha_0)]} = 0$

Phase locking at $\nu_0 = \pm\Omega$: $j_1 = -(j_{ac}/2i_0) \cos(\alpha_0 \pm \beta)$

Symmetric steps of total height $j_{\text{step}} = j_{ac}/\sqrt{\Omega^2 + 1}$

The minimal model, small second harmonic:

- integer steps at $\bar{v} = \pm\Omega$:

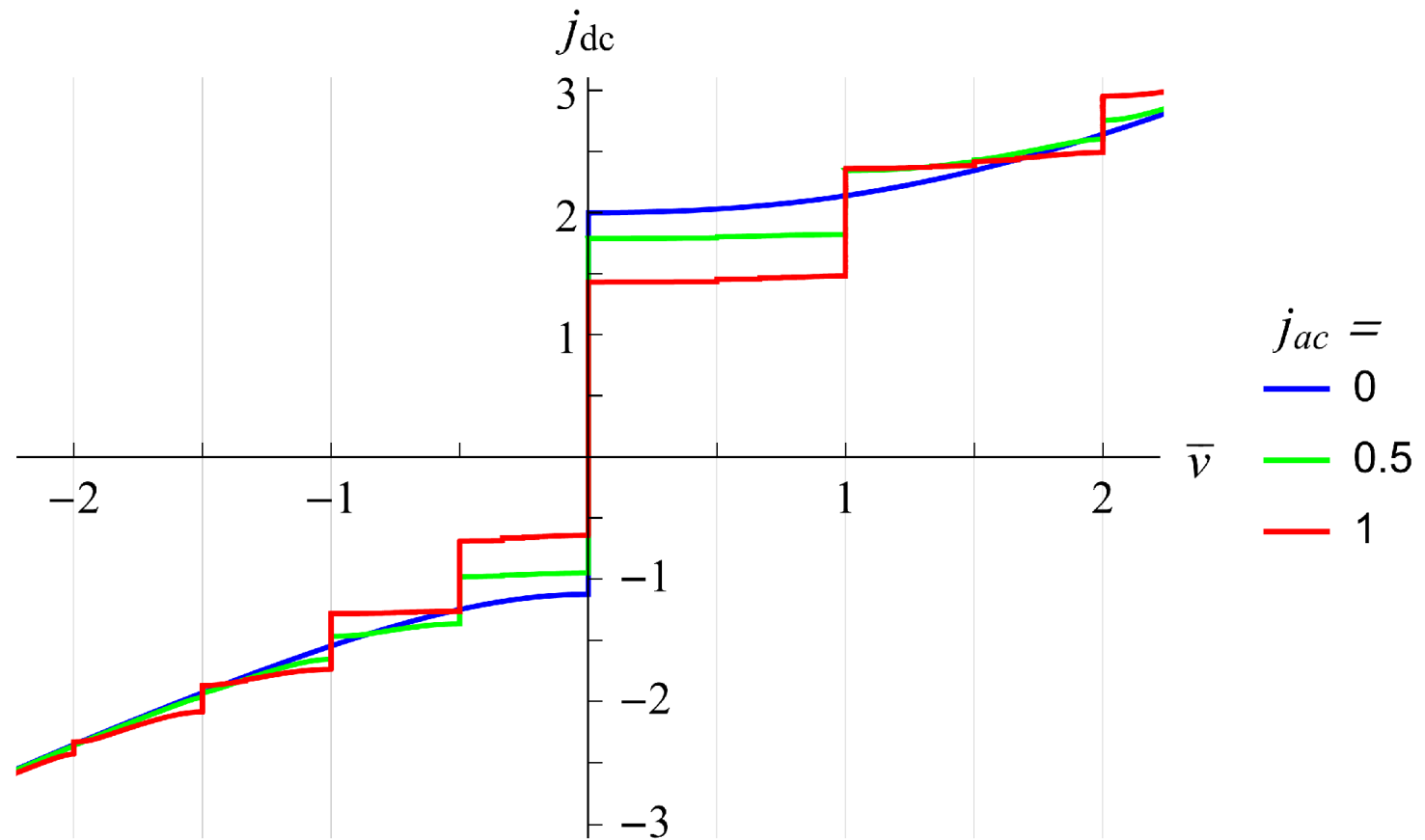
$$j_{\text{step}}^{(\pm 1)} = \frac{j_{ac}}{\sqrt{\Omega^2 + 1}} \left(1 \pm A\Omega \frac{2\Omega^4 + 3(\Omega^2 + 1) - 2\Omega(\Omega^2 + 1)^{3/2}}{\Omega^2 + 1} \sin \tilde{\phi} \right)$$

— asymmetry, JDE

- fractional steps at $\bar{v} = \pm\Omega/2$:

$$j_{\text{step}}^{(\pm 1/2)} = \frac{2Aj_{ac}}{3}\Omega \left[1 - \frac{\Omega^2}{2} + \frac{\Omega^3}{2\sqrt{\Omega^2 + 4}} \right]$$

Shapiro steps: numerical results



Diode effect: $j_c^+ \neq j_c^-$
 $j_{dc}(-\bar{v}) \neq -j_{dc}(\bar{v})$

Conclusions

Josephson diode effect in asymmetric SQUIDs with higher Josephson harmonics.

Minimal model: $J(\varphi) = \sin \varphi + A \sin(2\varphi - \tilde{\phi})$, at $A \sin \tilde{\phi} \neq 0$

- Asymmetric critical current: $I_c^+ \neq I_c^-$
- Asymmetric current-voltage characteristic: $I(-V) \neq -I(V)$
- Asymmetric integer Shapiro spikes (voltage-driven regime)
- Asymmetric integer and fractional Shapiro steps (current-driven regime),
$$\overline{V} = \left(\frac{n}{k}\right) \frac{\hbar\omega}{2e}$$
- Efficiency and polarity of the diode effect depend on the external magnetic flux

- ❑ Thanks for discussions: M.V. Feigel'man, P.M. Ostrovsky, V.V. Ryazanov
- ❑ Work supported by the Russian Science Foundation (Grant No. 21-42-04410)

Note added

PHYSICAL REVIEW LETTERS **129**, 267702 (2022)

Josephson Diode Effect in Supercurrent Interferometers

Rubén Seoane Souto,^{1,2} Martin Leijnse^{1,2} and Constantin Schrade¹

Superconducting Diode Effect in a Three-terminal Josephson Device

Mohit Gupta,¹ Gino V. Graziano,¹ Mihir Pendharkar,^{2,3} Jason T. Dong⁴,

Connor P. Dempsey,² Chris Palmstrøm,^{2,4,5} and Vlad S. Pribiag^{1*}

[arXiv:2206.08471](https://arxiv.org/abs/2206.08471)

